Nonlocal gravity and comparison with cosmological datasets

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Cosmology on Safari, Jan. 2015

based on

Jaccard, MM, Mitsou, PRD 2013, 1305.3034

MM, PRD 2014, 1307.3898

Foffa, MM, Mitsou, PLB 2014, 1311.3421

Foffa, MM, Mitsou, IJMPA 2014, 1311.3435

Kehagias and MM, JHEP 2014, 1401.8289

MM and Mancarella, PRD 2014, 1402.0448

Dirian, Foffa, Khosravi, Kunz, MM, JCAP 2014, 1403.6068

Dirian, Foffa, Kunz, MM, Pettorino, 1411.7692

the general idea: modify GR in the infrared using non-local terms

- motivation: explaining DE

  IR modification $\longrightarrow$ mass term?

- (local) massive gravity: Fierz-Pauli, dRGT, bigravity
  - significant progresses (ghost-free), still open issues

- our approach: mass term as coefficient of non-local terms non-locality emerges from fundamental local theories in many situations

- classically, when separating long and short wavelength and integrating out the short wave-length

  (e.g cosmological perturbation theory)

- in QFT, when computing the effective action that includes the effect of radiative corrections. This provides effective non-local field eqs for the vev of the fields

- UV divergences in curved space:

  - well understood (e.g. Birrel-Davies textbook, 1982)

  - renormalization of the UV divergences leaves finite non-local terms in the effective action. E.g.

$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2}\phi(\Box + m^2 + \xi R)\phi \right]$$

$$e^{iS_{\text{eff}}[g_{\mu\nu}]} = \int [d\phi] e^{iS[g_{\mu\nu},\phi]}$$

to order R^2:

$$S_{\text{eff}} = \int d^4x\sqrt{-g}\,[R + c_1 R^{\mu\nu\rho\sigma}\Gamma(\Box)R_{\mu\nu\rho\sigma} + c_2 R^{\mu\nu}\Gamma(\Box)R_{\mu\nu} + c_3 R\Gamma(\Box)R]$$

$$\Gamma(\Box) = \frac{2}{\epsilon} + \log 4\pi - \gamma_E - \log(\Box/\mu^2)$$

- related to running of coupling constant
- the imaginary parts describe particle production
- gives effects relevant only at large curvature

  possibly relevant for the Big-Bang singularity, not for dark energy

- IR effects in curved space are much less understood

interesting effects in deSitter for scalar fields with m << H

- super-Hubble fluctuations grow linearly in time

$$\langle \phi^2(t) \rangle \propto t$$

Starobinsky 1986,...

- and saturate at $\langle \phi^2(t) \rangle \propto H^4/m^2$

- adding a $\lambda\phi^4$ interaction, because of IR divergences, the actual loop expansion parameter is $\lambda H^2/m^2$

  breakdown of perturbation theory (e.g. Polyakov 1986,2012 Burgess et al 2010)

- strong IR divergencies in inflationary correlation functions

(e.g. review Seery 2010)

A related question: what is the EFT theory at the horizon scale?

- the standard Wilsonian EFT is not the appropriate framework:

  the IR and UV sector exchange energy because of the time-dependent background

- EFT of open system
  - e.g. Starobinsky stochastic eq. for super-Hubble modes
  - no effective low-energy action. Rather stochastic and dissipative eqs for the IR modes
  - in general this produces non-Markovian dynamics -> non-locality in time

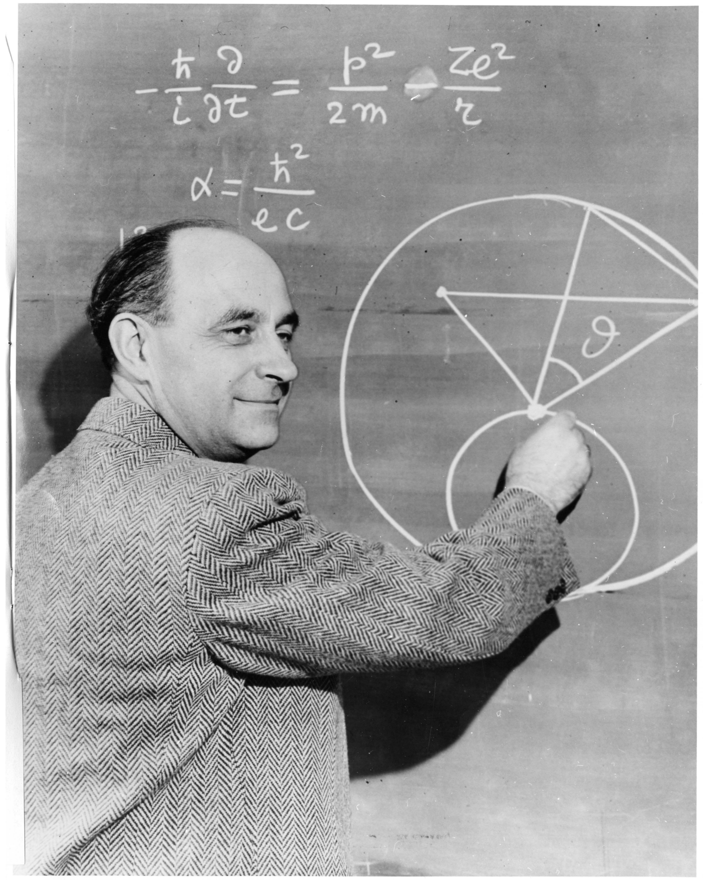
Burgess,Holman, Tasinato and Williams 1408.5002

Agon,Balasubramanian,Kasko,Lawrence 1412.3148 bottomline:

- IR effect in curved space are not well understood
- potentially relevant for understanding dark energy
- generically, these effects should generate non-local terms in effective actions

a phenomenological attitude: which effective nonlocal theories can give a meaningful cosmology?

- top-down approach: find the correct fundamental theory and the mechanism that generates nonlocality
- bottom-up: find first the correct effective theory



- e.g. Standard Model vs Fermi theory
  - start from the fundamental YM theory
  - or understand which terms correctly describe weak interaction at low energies

$$\text{e.g. } (\bar\psi\psi)^2,\ (\bar\psi\gamma_5\psi)^2,\ (\bar\psi\gamma_\mu\psi)^2,$$
$$\ldots [\bar\psi\gamma_\mu(1-\gamma_5)\psi]^2,$$

some sources of inspiration: a locality / gauge-invariance duality for massive gauge fields

- Proca theory for massive photons

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m_\gamma^2 A_\mu A^\mu - j_\mu A^\mu \right]$$

$$\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = j^\nu \quad \rightarrow \left\{ \begin{array}{r} m_\gamma^2\, \partial_\nu A^\nu = 0 \\ (\Box - m_\gamma^2) A^\mu = 0 \end{array} \right.$$

- non-local formulation (Dvali 2006)

Stueckelberg trick: $A_\mu \rightarrow A_\mu + \frac{1}{m_\gamma} \partial_\mu \varphi$

we add one field and we gain a gauge symmetry

$$A_\mu \rightarrow A_\mu - \partial_\mu \theta\,, \quad \varphi \rightarrow \varphi + m_\gamma \theta$$

$$S = \int d^4x \, \left[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2} m_\gamma^2 A_\mu A^\mu - \tfrac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - m_\gamma A^\mu \partial_\mu \varphi - j_\mu A^\mu \right]$$

$$\partial_\mu F^{\mu\nu} = m_\gamma^2 A^\nu + m_\gamma \partial^\nu \varphi + j^\nu \, ,$$
$$\Box \varphi + m_\gamma \partial_\mu A^\mu = 0 \, .$$

If we choose the unitary gauge ϕ=0 we get back to the original formulation of Proca theory (and loose the gauge sym because of gauge fixing).

Instead, keep the gauge sym explicit and integrate out ϕ using its own equation of motion:

$$\varphi(x) = -m_\gamma \Box^{-1} (\partial_\mu A^\mu)$$

Substituting in the eq of motion for A^ν :

$$\left(1 - \frac{m_\gamma^2}{\Box}\right) \partial_\mu F^{\mu\nu} = j^\nu$$

or

$$(\Box - m_\gamma^2) A^\nu = \left(1 - \frac{m_\gamma^2}{\Box}\right) \partial^\nu \partial_\mu A^\mu + j^\nu$$

we have explicit gauge invariance for the massive theory, at the price non-locality

- a sort of duality between explicit gauge-invariance and explicit locality
- we can fix the gauge $\partial_\mu A^\mu = 0$ and the non-local term disappears (and we are back to Proca eqs.)
- with hindsight, the Stueckelberg trick was not needed

possible implementations of this idea in GR

in QED, we found that a massive deformation of the theory is obtained replacing

$$\partial_\mu F^{\mu\nu} = j^\nu \quad \rightarrow \quad \left(1 - \frac{m^2}{\Box}\right) \partial_\mu F^{\mu\nu} = j^\nu$$

- for gravity, a first guess for a massive deformation of GR could be

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \rightarrow \quad \left(1 - \frac{m^2}{\Box_g}\right) G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

(Arkani-Hamed, Dimopoulos, Dvali and Gabadadze 2002)

however this is not correct since $\nabla^\mu (\Box_g^{-1} G_{\mu\nu}) \neq 0$

We would lose energy-momentum conservation.

- to preserve energy-momentum conservation:

$$G_{\mu\nu} - m^2(\Box^{-1}G_{\mu\nu})^T = 8\pi G T_{\mu\nu}$$

(Jaccard,MM, Mitsou, 2013)

however, instabilities in the cosmological evolution

(Foffa,MM, Mitsou, 2013)

- $$G_{\mu\nu} - m^2(g_{\mu\nu}\Box^{-1}R)^T = 8\pi G T_{\mu\nu}$$

(MM 2013)

stable cosmological evolution!

- last twist

$$S_{\rm NL} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - m^2 R\frac{1}{\Box^2}R\right]$$

(MM and M.Mancarella, 2014)

- So, we interpret our non-local eqs as a classical, effective equation, derived from a more fundamental local theory by a classical or quantum averaging
- any problem of quantum vacuum stability can only be addressed in this fundamental theory
- the theory $S_{\rm NL} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - m^2 R \frac{1}{\Box^2} R \right]$

  could be the truncation of the correct effective theory
- the theory $G_{\mu\nu} - m^2 (g_{\mu\nu} \Box^{-1} R)^T = 8\pi G T_{\mu\nu}$

  could be an example of resummation
- our general question: which effective nonlocal theories give a meaningful cosmology?

Absence of vDVZ discontinuity and of a strong coupling regime

A. Kehagias and MM 2014

- write the eqs of motion of the non-local theory in spherical symmetry:

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

- for mr <<1: low-mass expansion

- for r>>r_S: Newtonian limit (perturbation over Minowski)

- match the solutions for $r_S << r << m^{-1}$ (this fixes all coefficients)

- result: for $r \gg r_s$

$$A(r) = 1 - \frac{r_S}{r}\left[1 + \frac{1}{3}(1-\cos mr)\right]$$

$$B(r) = 1 + \frac{r_S}{r}\left[1 - \frac{1}{3}(1-\cos mr - mr\sin mr)\right]$$

for $r_s \ll r \ll m^{-1}$: $A(r) \simeq 1 - \frac{r_S}{r}\left(1 + \frac{m^2r^2}{6}\right)$

the limit $m \to 0$ is smooth !

By comparison, in massive gravity the same computation gives

$$A(r) = 1 - \frac{4}{3}\frac{r_S}{r}\left(1 - \frac{r_S}{12m^4r^5}\right)$$

vDVZ discontinuity

breakdown of linearity below $r_V = (r_s/m^{\wedge}4)^{1/5}$

Cosmological consequences.

- consider $$S_{\rm NL} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - m^2 R\frac{1}{\Box^2}R\right]$$

  define $$U = -\Box^{-1}R\,, \quad S = -\Box^{-1}U$$

  NB: auxiliary non-dynamical fields! U=0 if R=0. It is not the same as a scalar-tensor theory

- in FRW we have 3 variables: H(t), U(t), W(t)=H^2(t)S(t).

  define $x=\ln a(t)$, $h(x)=H(x)/H_0$,

  $\gamma=(m/3H_0)^2$ $\zeta(x)=h'(x)/h(x)$

$$h^2(x) = \Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y(U, U', W, W')$$
$$U'' + (3+\zeta)U' = 6(2+\zeta)$$
$$W'' + 3(1-\zeta)W' - 2(\zeta' + 3\zeta - \zeta^2)W = U$$

- there is an effective DE term, with

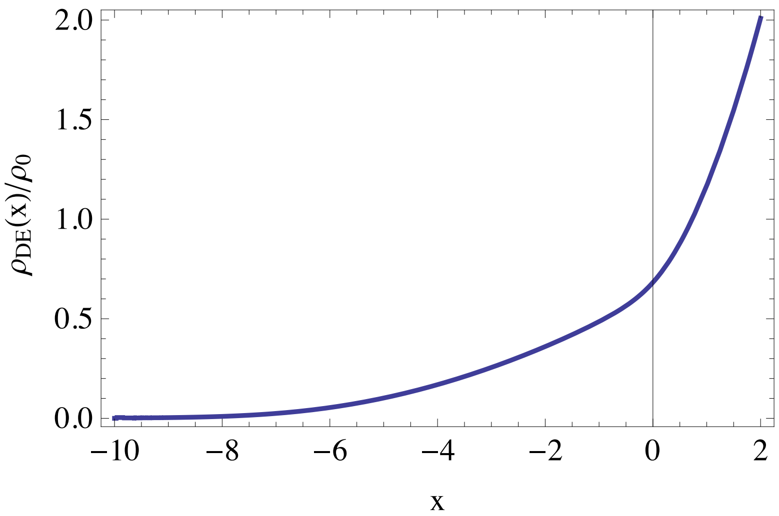
$$\rho_{\rm DE}(x) = \rho_0 \gamma Y(x) \qquad\qquad \rho_0 = 3H_0^2/(8\pi G)$$

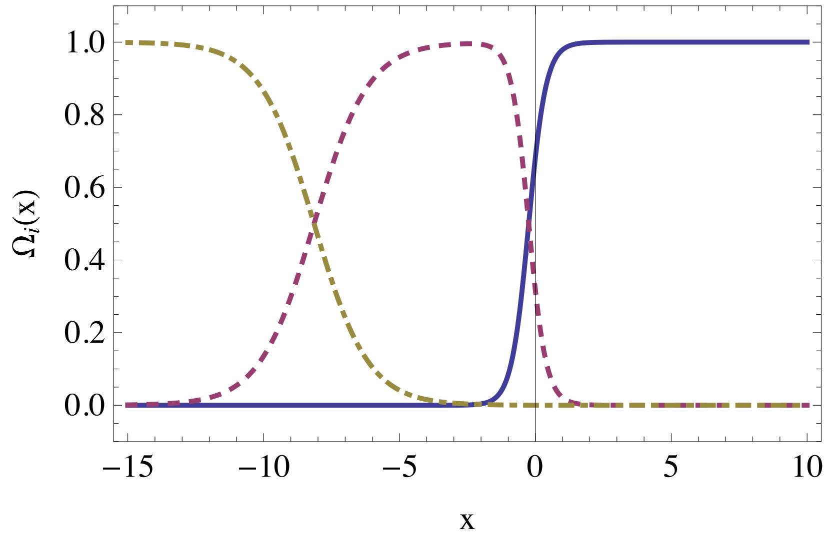
- define w_{DE} from

$$\dot\rho_{\rm DE} + 3(1+w_{\rm DE})H\rho_{\rm DE} = 0$$

- the model has the same number of parameters as ΛCDM, with $\Omega_\Lambda \leftrightarrow \gamma$.

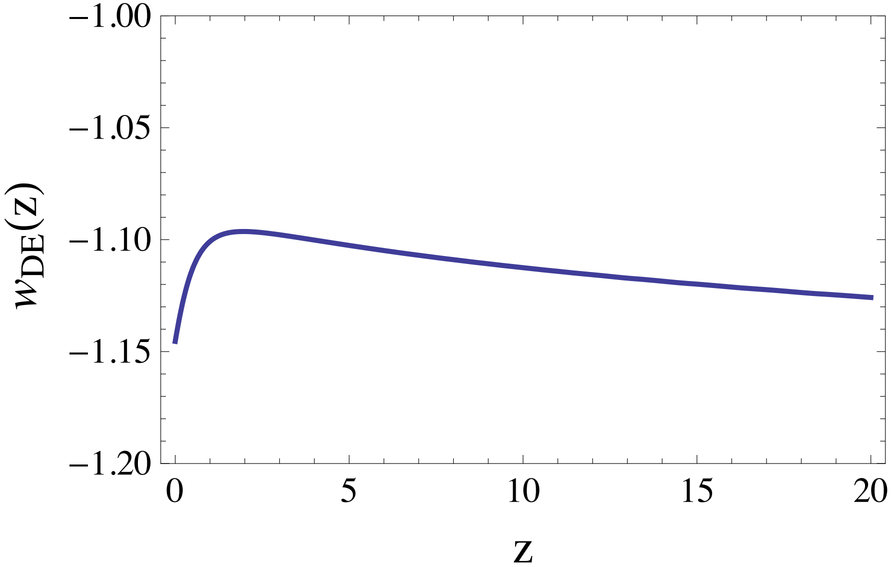
- results:





- Fixing γ = 0.0089.. (m=0.28 H_0) we reproduce Ω_{DE}=0.68

- having fixed γ we get a pure prediction for the EOS:



fit $w(a)=w_0+(1-a)\,w_a$

in the region $0< z <1.6$

$w_0= -1.14$, $w_a=0.08$

on the phantom side ! general consequence of

$$\dot\rho_{\rm DE}+3(1+w_{\rm DE})H\rho_{\rm DE}=0$$

together with $\rho>0$ and $d\rho/dt>0$

The RT model $G_{\mu\nu}-m^2(g_{\mu\nu}\Box^{-1}R)^T=8\pi G T_{\mu\nu}$

gives $w_0= -1.04$, $w_a=-0.02$

warning. This is not wCDM !!!

Cosmological perturbations

Y. Dirian, S. Foffa, N. Khosravi, M. Kunz, MM
1403.6068

- well-behaved?

- consistent with data?

- Comparison with ΛCDM

An aside: the Deser-Woodard non-local model

with phenomenological motivations similar to ours, has been proposed a model of the form

$$S = \int d^4x\sqrt{-g}\,\left[R + Rf(\Box^{-1}R)\right]$$

Deser and Woodard 2007

much activity on ``reconstruction'' of f(R):

$$f(X) = a_1[\tanh(a_2 + a_3 X + a_4 X^2 + a_5 X^3) - 1]$$

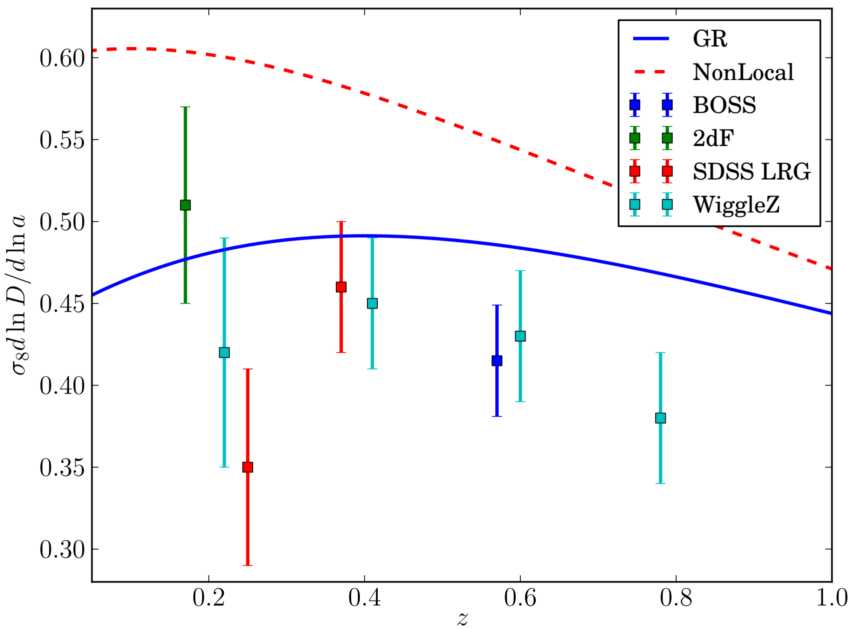
- not predictive at the background level: chosen to mimic ΛCDM
- by comparison, our model is

$$S = \int d^4x\,\left[R - m^2 f(\Box^{-1}R)\right] \qquad f(X) = X^2$$

after fixing the background evolution in this way, one can comoute cosmological perturbations in the Deser-Woodard model, and compare with data

Deser-Woodard model ruled out at the 8σ level by structure formation

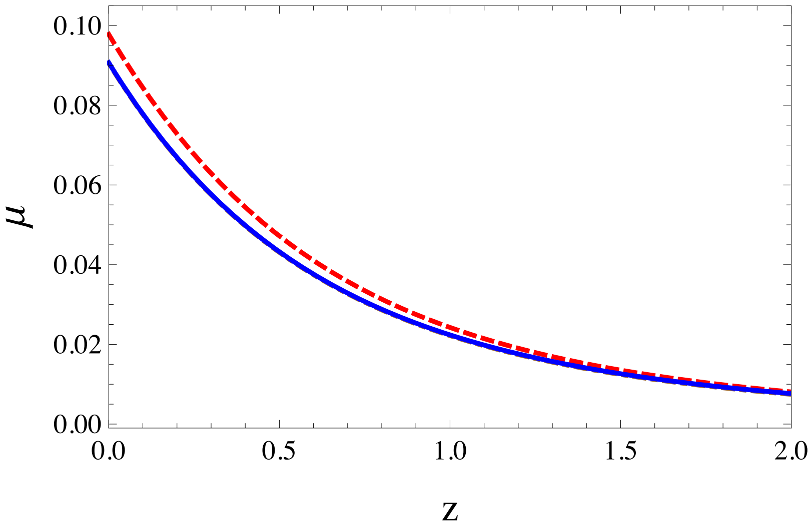
Dodelson and Park 2013



how our model performs??

- the perturbations are well-behaved and differ from ΛCDM at a few percent level

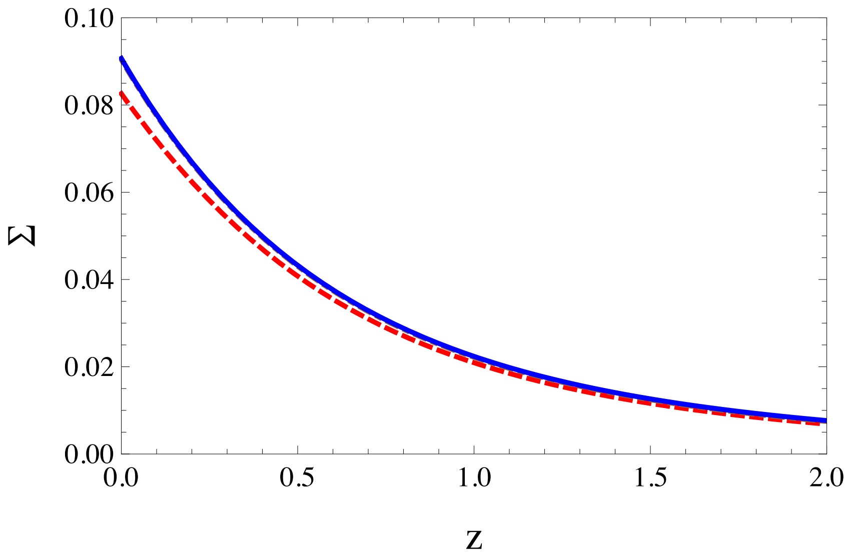
$$\Psi = [1+\mu(a;k)]\Psi_{\rm GR}$$
$$\Psi - \Phi = [1+\Sigma(a;k)](\Psi-\Phi)_{\rm GR}$$



- deviations at z=0.5 of order 4%

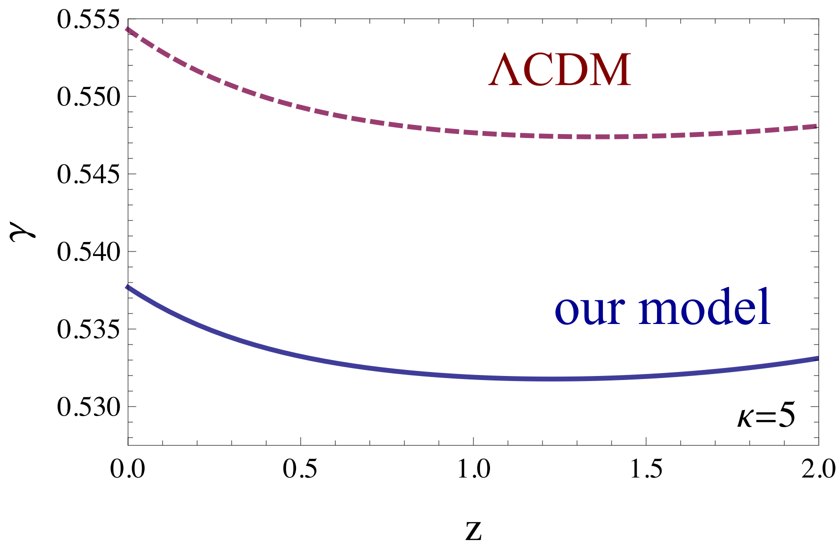
- consistent with data: CFHTLenS gives ΔΨ/Ψ=0.05±0.25 (Simpson et al 1212.3339)

Lensing: again deviations at 4% level



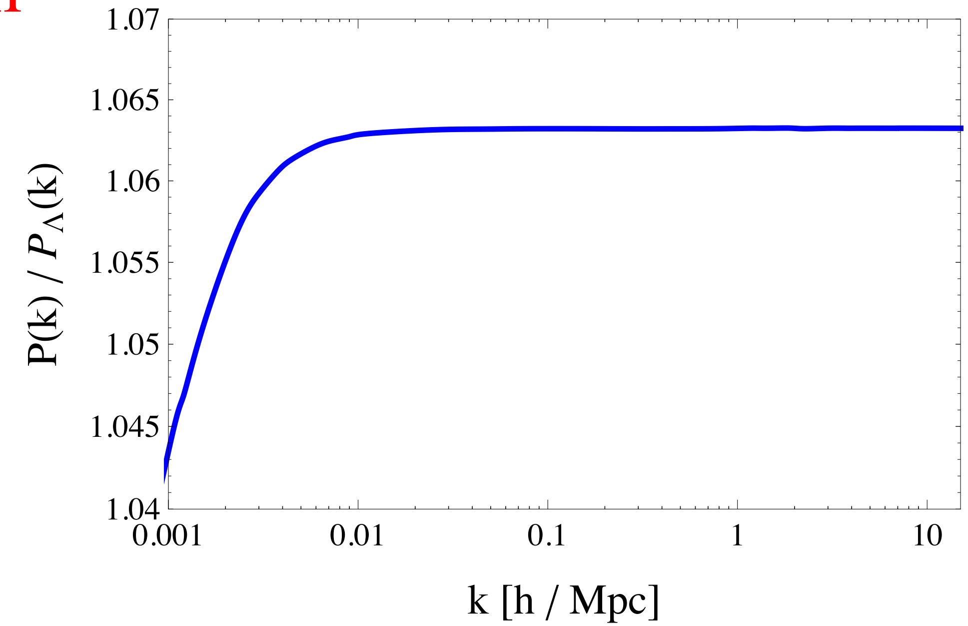
growth index:

$$\frac{d\log\delta_M(a;k)}{d\ln a} = [\Omega_M]^{\gamma(z;k)}$$

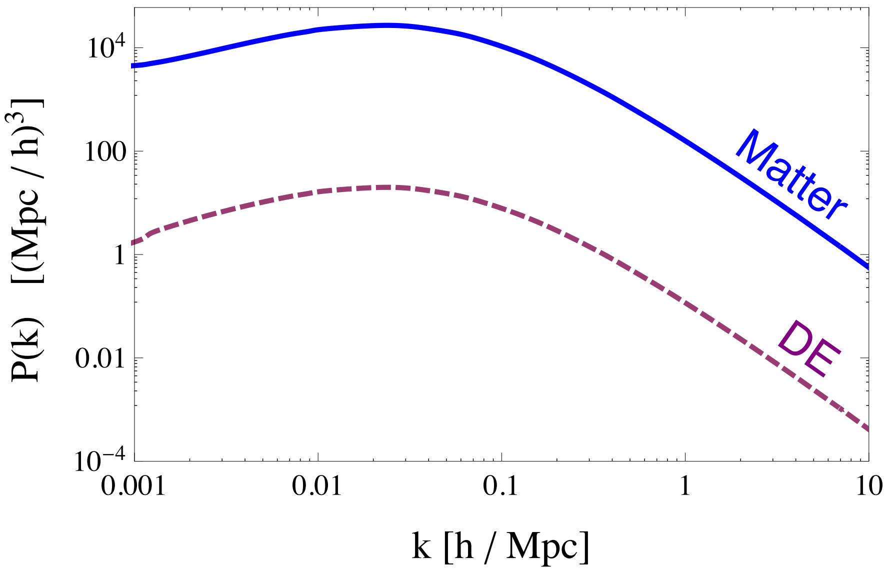


- linear power spectrum

matter power spectrum
compared to ΛCDM



DE clusters but its linear
power spectrum is small
compared to that of matter



Boltzmann code analysis and comparison with data

Dirian, Foffa, Kunz, MM, Pettorino, 1411.7692

- CMB data from the Planck 2013 data release, type-Ia supernovae from JLA and BAO data from BOSS

- we modified the CLASS code and use Montepython MCMC

- we vary $\omega_b = \Omega_b h_0^2, \, \omega_c = \Omega_c h_0^2, \, H_0, \, A_s, \, n_s, \, z_{\rm re}$

  In ΛCDM, Ω_Λ is a derived parameter, fixed by the flatness condition. Similarly, in our model the mass parameter m^2 is a derived parameter, fixed again from $\Omega_{tot}=1$

we have the same number of free parameters as in ΛCDM

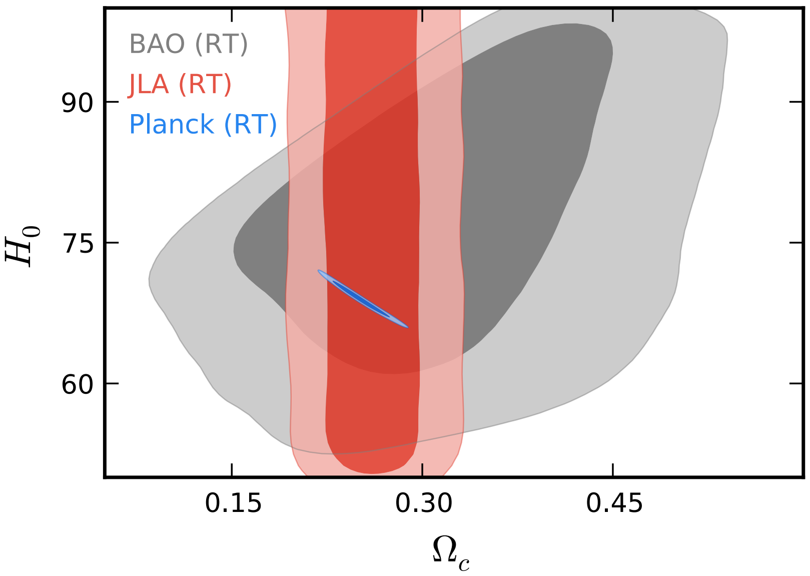
- # Results

Param	ΛCDM	$g_{\mu\nu}\Box^{-1}R$	$R\Box^{-2}R$
$100\ \omega_b$	$2.201^{+0.028}_{-0.029}$	$2.204^{+0.028}_{-0.03}$	$2.207^{+0.029}_{-0.029}$
ω_c	$0.1194^{+0.0027}_{-0.0026}$	$0.1195^{+0.0026}_{-0.0028}$	$0.1191^{+0.0027}_{-0.0028}$
H_0	$67.56^{+1.2}_{-1.3}$	$68.95^{+1.3}_{-1.3}$	$71.67^{+1.5}_{-1.5}$
$10^9 A_s$	$2.193^{+0.052}_{-0.06}$	$2.194^{+0.048}_{-0.062}$	$2.198^{+0.053}_{-0.059}$
n_s	$0.9625^{+0.0072}_{-0.0074}$	$0.9622^{+0.007}_{-0.0081}$	$0.9628^{+0.0074}_{-0.0073}$
z_{re}	$11.1^{+1.1}_{-1.1}$	$11.1^{+1.1}_{-1.2}$	$11.16^{+1.2}_{-1.1}$
$\chi^2_{\rm min}$	9801.7	9801.3	9800.1

Table 1: *Planck* CMB data only.

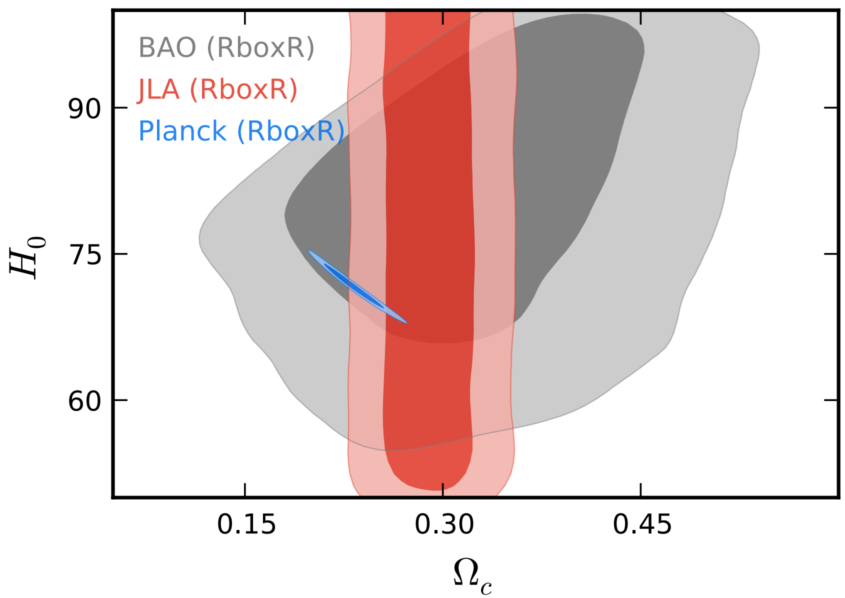
Param	ΛCDM	$g_{\mu\nu}\Box^{-1}R$	$R\Box^{-2}R$
$100\ \omega_b$	$2.215^{+0.025}_{-0.025}$	$2.207^{+0.024}_{-0.025}$	$2.197^{+0.024}_{-0.025}$
ω_c	$0.1175^{+0.0015}_{-0.0014}$	$0.1188^{+0.0014}_{-0.0014}$	$0.1204^{+0.0014}_{-0.0013}$
H_0	$68.43^{+0.61}_{-0.69}$	$69.3^{+0.68}_{-0.66}$	$70.94^{+0.74}_{-0.7}$
$10^9 A_s$	$2.199^{+0.055}_{-0.062}$	$2.196^{+0.052}_{-0.065}$	$2.192^{+0.051}_{-0.061}$
n_s	$0.9668^{+0.0055}_{-0.0054}$	$0.9636^{+0.0052}_{-0.0055}$	$0.9599^{+0.0052}_{-0.0051}$
z_{re}	$11.33^{+1.1}_{-1.1}$	$11.18^{+1.1}_{-1.2}$	$11.00^{+1.1}_{-1.2}$
$\chi^2_{\rm min}$	10485.5	10485.0	10488.7

Planck+JLA+BAO

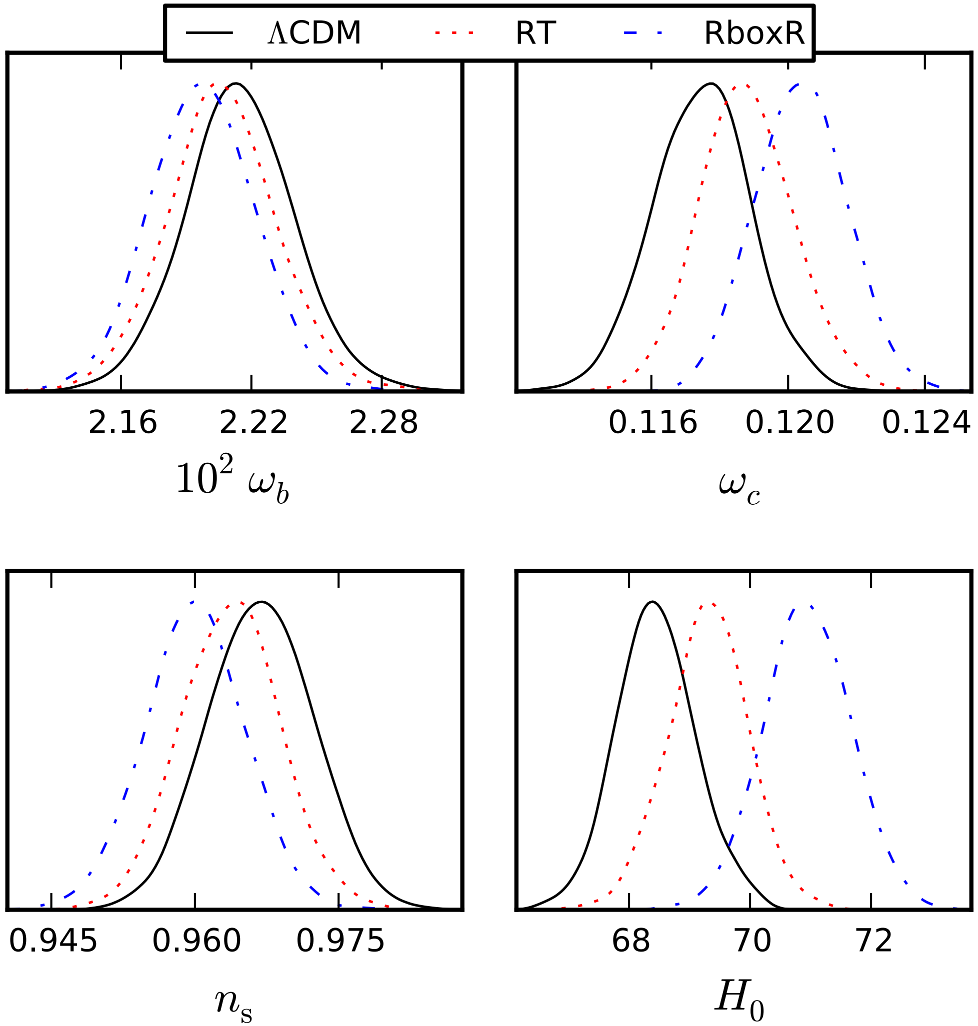


The RT model works perfectly well

(visually similar plot for ΛCDM)



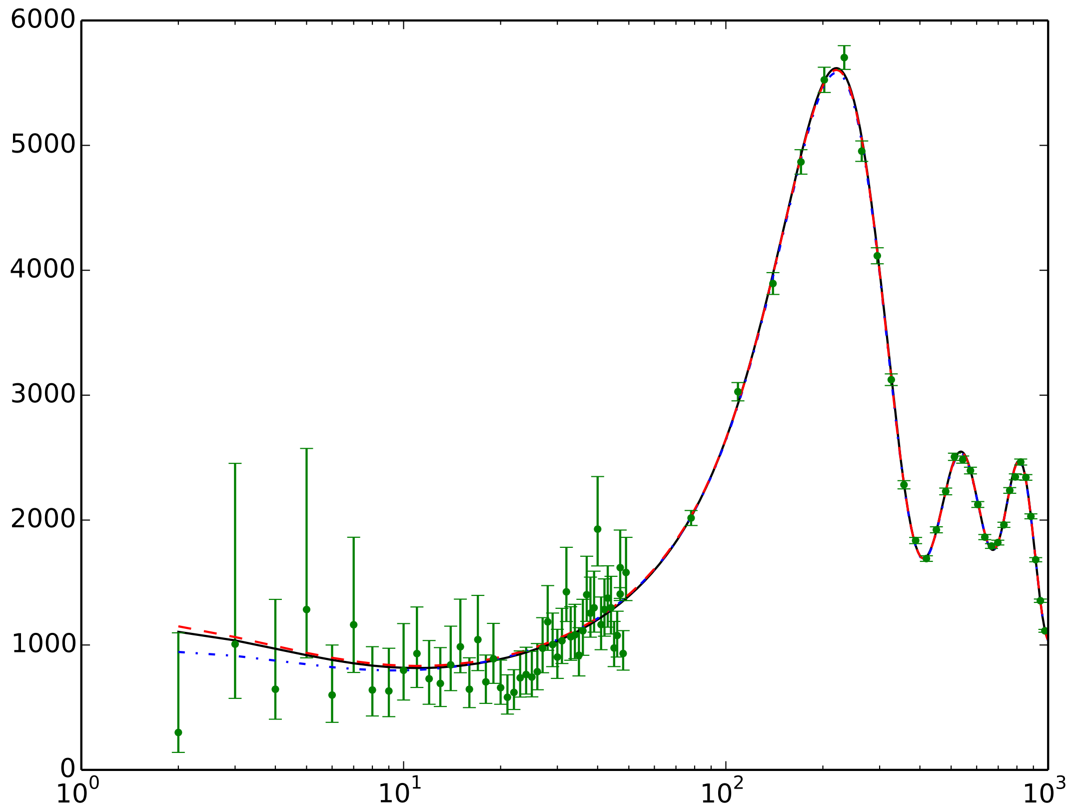
The RboxR model has a slight (2σ) tension between CMB and SN



excellent agreement with local H_0 measurements.

Latest revised value after correcting for star formation bias
$H_0 = 70.6 \pm 2.6$
(Rigault et al 1412.6501)

using Planck+JLA+BAO



LCDM and RT model almost indistinguishable
RboxR (blue dot-dashed) lower at low multipoles

Conclusions

- we have an interesting IR modification of GR
- at the phenomenological level, it works very well
  - solar system tests OK
  - generates dynamically a dark energy
  - cosmological perturbations work well
  - passes tests of structure formation
  - comparison with CMB,SNe,BAO with modified Boltzmann code ok
  - higher value of H0

It is the only existing model, with the same number of parameters as ΛCDM, which is competitive with ΛCDM from the point of view of fitting the data

- sufficiently close to ΛCDM to be consistent with existing data, but distinctive prediction that can be clearly tested in the near future

  - phantom DE eq of state: w(0)= - 1.14 (RboxR) (or -1.04 RT) + a full prediction for w(z)
    - DES Δw=0.03
    - EUCLID Δw=0.01
  - linear structure formation

    $$\mu(a) = \mu_s a^s \rightarrow \mu_{\mathbf{s}} = \mathbf{0.09}, \mathbf{s} = \mathbf{2}$$

    - Forecast for EUCLID, Δμ=0.01
  - non-linear structure formation: 10% more massive halos

    Barreira, Li, Hellwing, Baugh, Pascoli 2014
  - lensing: deviations at a few %

For the future

- At the phenomenological level: comparison with the more accurate data expected in the near future

  (at least, a useful model against which compare ΛCDM)

- At the fundamental level: understand the origin of the non-local term (with the advantage that we now know what we should be looking for)

Thank you!

Degrees of freedom

$$S_{\rm NL} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\left[R - m^2 R\frac{1}{\Box^2}R\right]$$

- define $U = -\Box^{-1}R\,, \quad S = -\Box^{-1}U$
- the eqs. $\Box U = -R, \quad \Box S = -U$

  do not describe radiative d.o.f !

$$-\Box^{-1}R = U_{\rm hom}(x) - \int d^4x'\,\sqrt{-g(x')}\,G(x;x')R(x')$$

The homogeneous solution is fixed by the definition of i.e. by the def of the non-local theory.

It is not a free Klein-Gordon field !

- linearize the eqs of motion. Scalar sector:

$$h_{00} = 2\Psi, \quad h_{0i} = 0, \quad h_{ij} = 2\Phi\delta_{ij}$$

$$\begin{aligned}
\nabla^2 \left[\Phi - (m^2/6)S\right] &= -4\pi G\rho \\
\Phi - \Psi - (m^2/3)S &= -8\pi G\sigma \\
(\Box + m^2)U &= -8\pi G(\rho - 3P) \\
\Box S &= -U
\end{aligned}$$

Φ and Ψ remain non-radiative!

In contrast, in massive gravity with FP mass term $(\Box - m^2)\Phi = 0$
and with generic mass there is a $(\Box\Phi)^2$ in the action (ghost)

U and S are non-radiative despite the KG operator.

No radiative d.o.f. in the scalar sector !

A technical point

- we have $\Box_{\rm ret}^{-1}$ directly in the EoM (rather than in the solution). This EoM cannot come from the variation of a Lagrangian. E.g.

$$\frac{\delta}{\delta\phi(x)}\int dx'\phi(x')(\Box^{-1}\phi)(x') = \frac{\delta}{\delta\phi(x)}\int dx'dx''\phi(x')G(x',x'')\phi(x'')$$
$$= \int dx'[G(x,x') + G(x',x)]\phi(x')$$

- we can repalce $\Box^{-1} \to \Box_{\rm ret}^{-1}$ after the variation, as a formal trick to get the EoM from a Lagrangian.

Deser-Waldron 2007,
Barvinski 2012

However, any connection to the QFT described by this Lagrangian is lost.