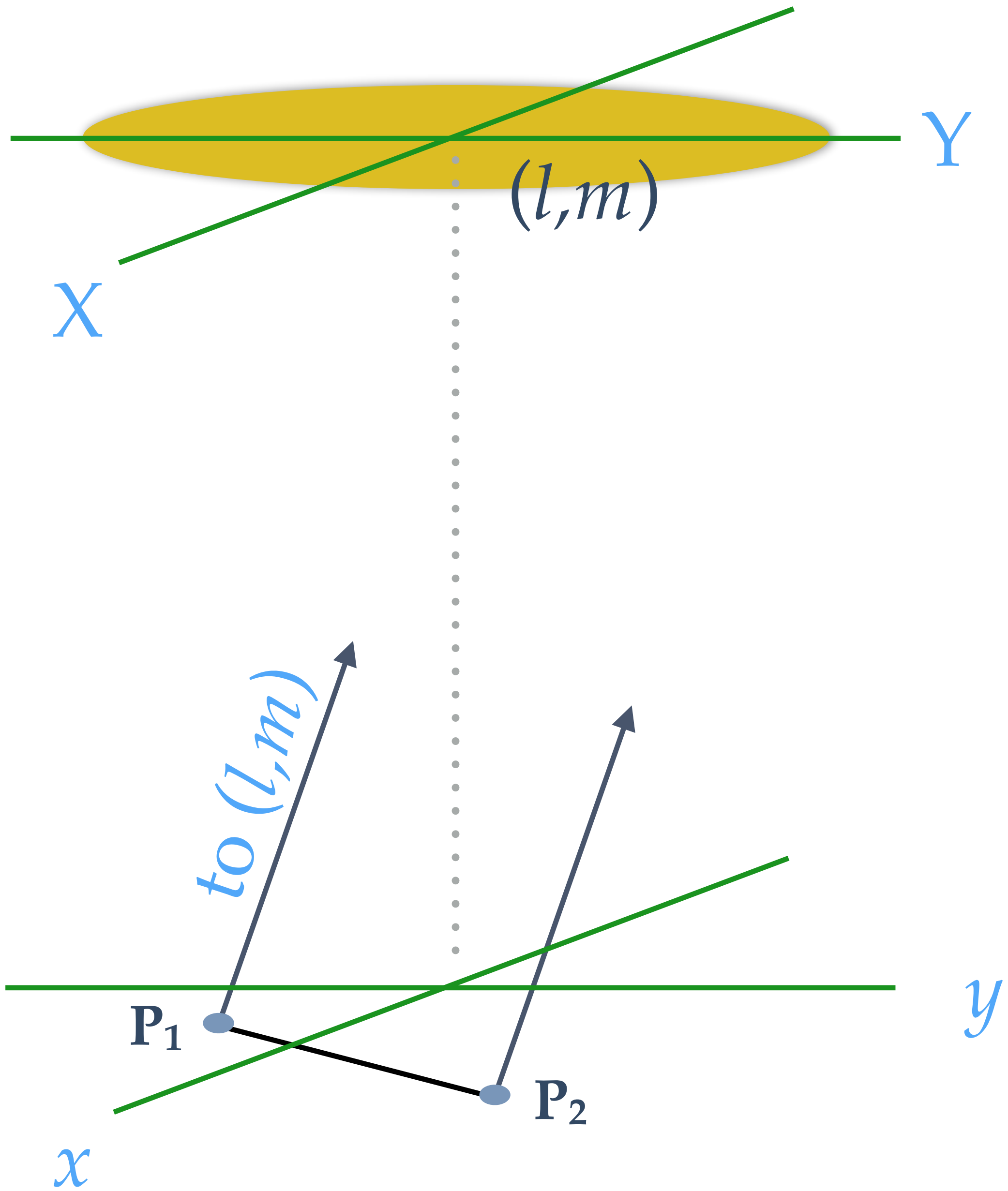
Fourier Transforms in Radio Astronomy

Kavilan Moodley, UKZN

Slides taken from N Gupta’s lectures: SKA School 2013

van-Cittert Zernike theorem

Extended, quasi-monochromatic, incoherent source



Field at P_1 and P_2 due to an element at (l,m):

$$E_1 = \mathbb{E}(l,m,t-\frac{R_1}{c})\frac{exp(-j2\pi\nu(t-\frac{R_1}{c}))}{R_1}$$

$$E_2 = \mathbb{E}(l,m,t-\frac{R_2}{c})\frac{exp(-j2\pi\nu(t-\frac{R_2}{c}))}{R_2}$$

(Thompson, Moran & Swenson)

van-Cittert Zernike theorem

The complex correlation at P_1 and P_2 for zero time offset:

$$< E_1(l,m,t) * E_2(l,m,t) > = < \mathbb{E}(l,m,t-\frac{R_1}{c}) * \mathbb{E}^*(l,m,t-\frac{R_2}{c}) > \frac{exp(-j2\pi\nu(t-\frac{R_1}{c}))}{R_1} \frac{exp(\ \ j2\pi\nu(t-\frac{R_2}{c}))}{R_2}$$

$$= < \mathbb{E}(l,m,t) * \mathbb{E}^*(l,m,t-\frac{R_2-R_1}{c}) > \frac{exp[j2\pi\nu(R_1-R_2)/c]}{R_1 R_2}$$

small wrt receiver BW^{-1}

$I(l,m)$

$$V_{12}(u,v,0) = \int \frac{I(l,m) exp[j2\pi\nu(R_1-R_2)/c]}{R_1 R_2} ds$$

$u = (x_2 - x_1)\nu/c$

$v = (y_2 - y_1)\nu/c$

$(R_2 - R_1) = (ul + vm)c/\nu$

$R_2 \simeq R_1 = R$

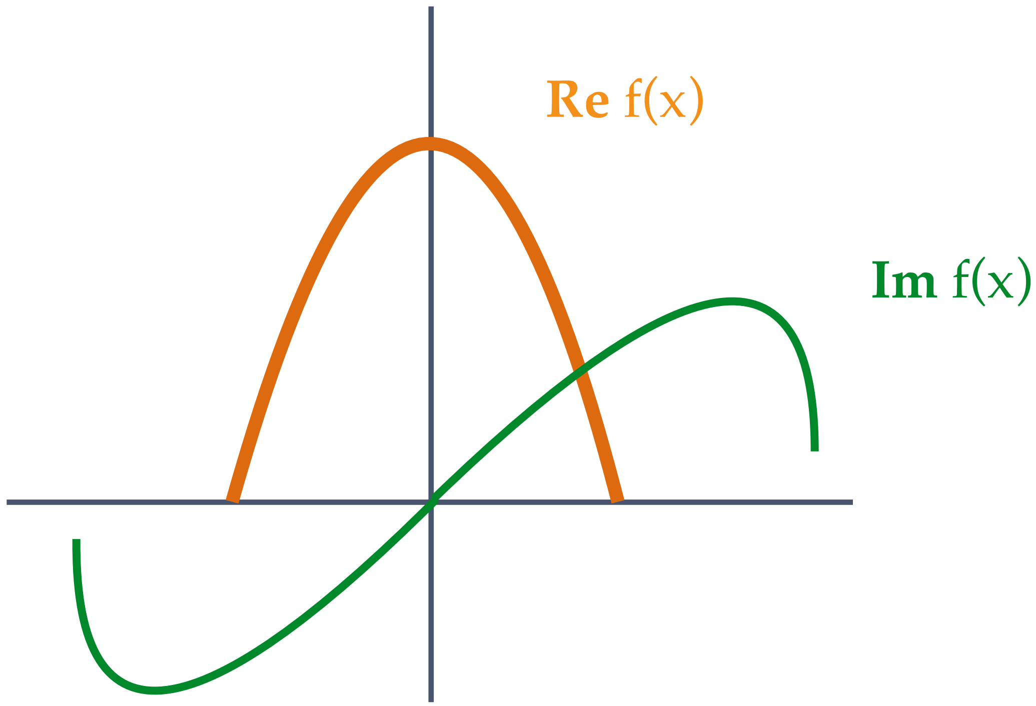
$$V_{12}(u,v,0) = \int\int I(l,m) e^{j2\pi(ul+vm)} dl dm$$

$V(u,v)$ ⟷ $I(l,m)$

(Thompson, Moran & Swenson)

I(l,m) is real; V(u,v) is Hermitian.

Hermitian function



$$f(x) = f^*(-x)$$
$$f(x) = E(x) + i\,O(x)$$

Real part is even;
Imag part is odd.

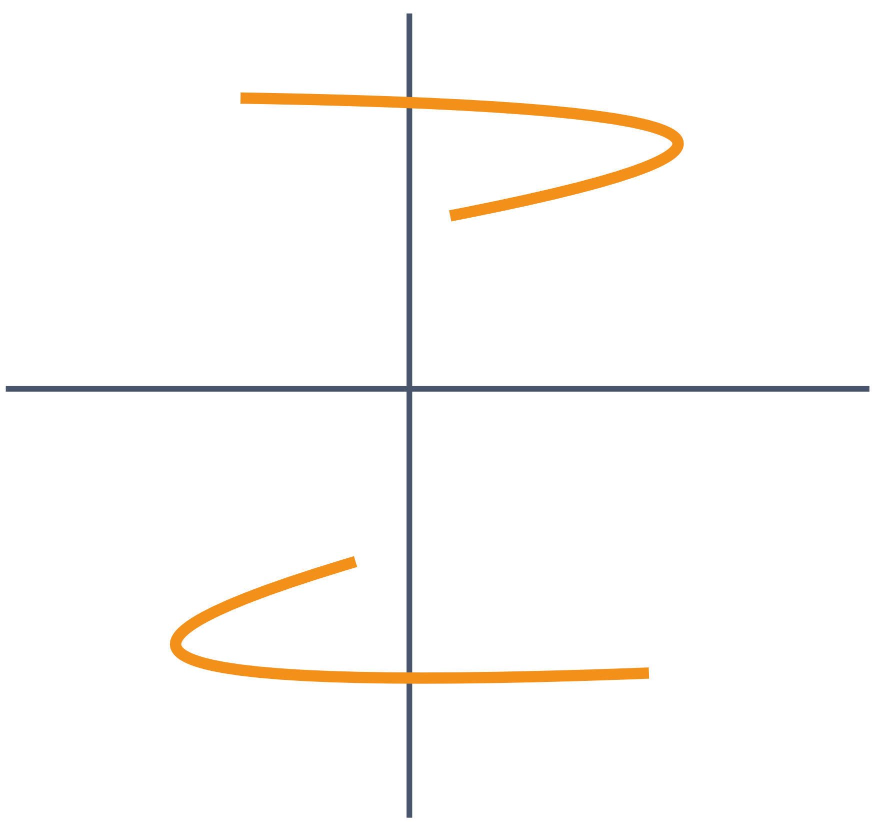
Since V(u,v) is hermitian we measure only half of the (u,v)-plane and fill the other half with the complex conjugates.

$$\mathbf{V(u,v) = V^*(-u,-v)}$$

$$\mathbf{V}(u,v) \overset{\text{FT}}{\longleftrightarrow} \mathbf{I}(l,m)$$

I(l,m) is real; V(u,v) is Hermitian.

(u,v) -plane



(u,v) tracks as ellipses

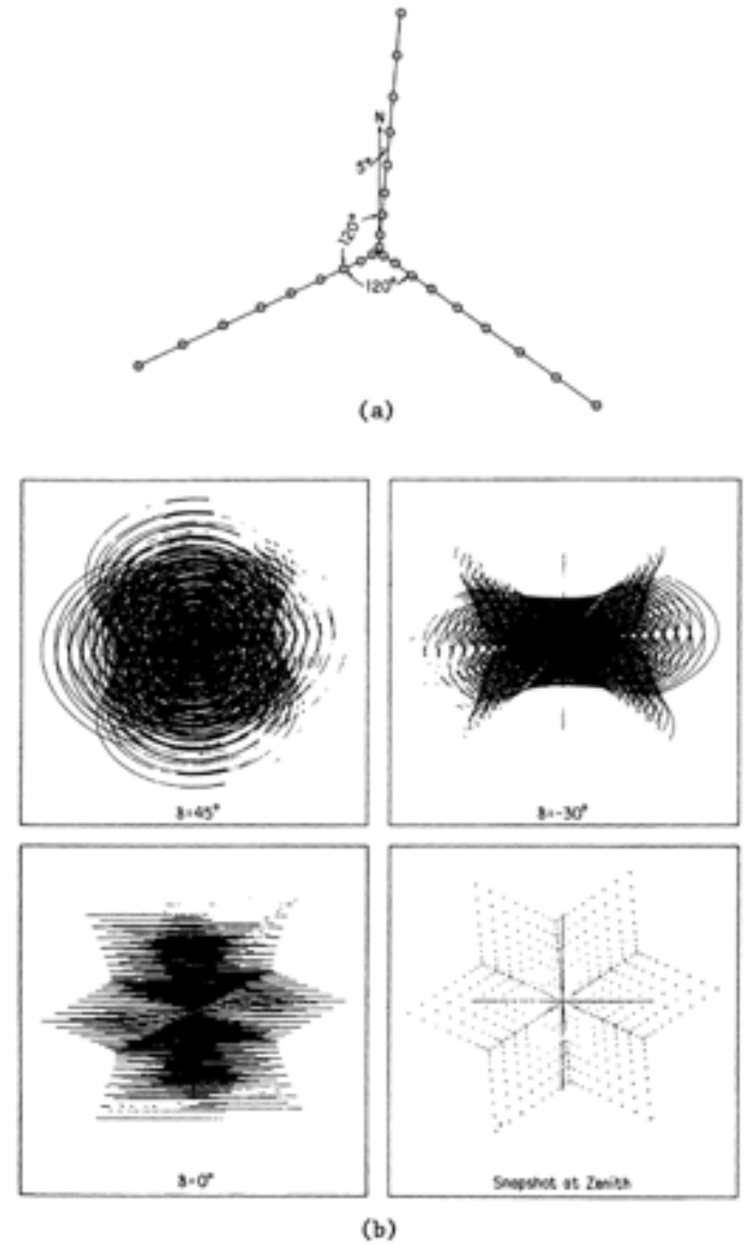
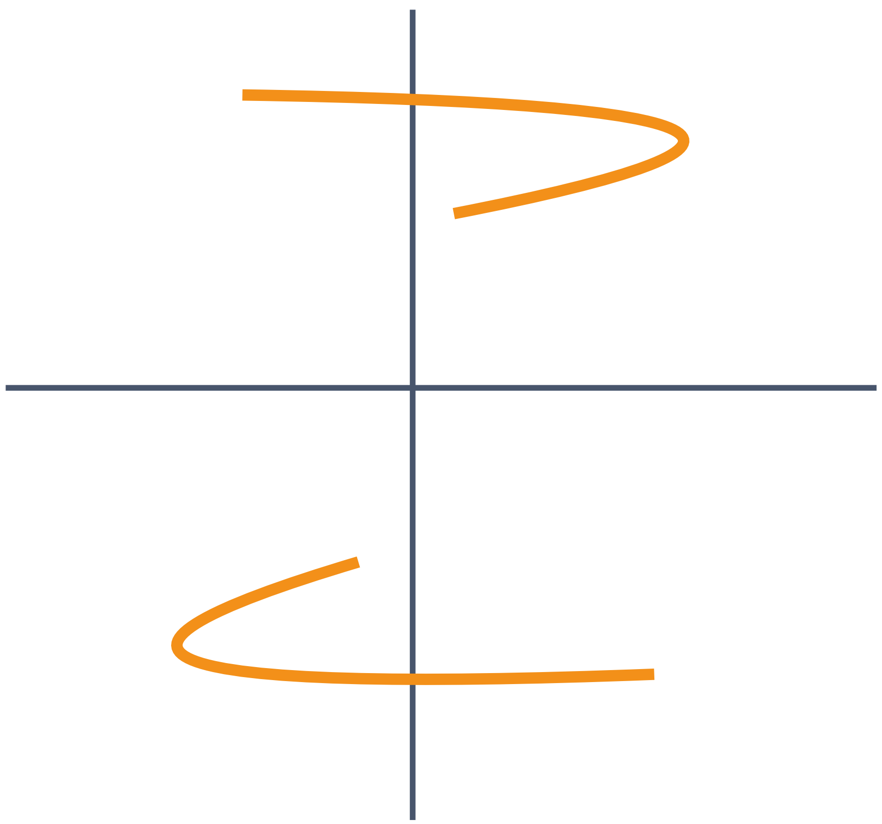


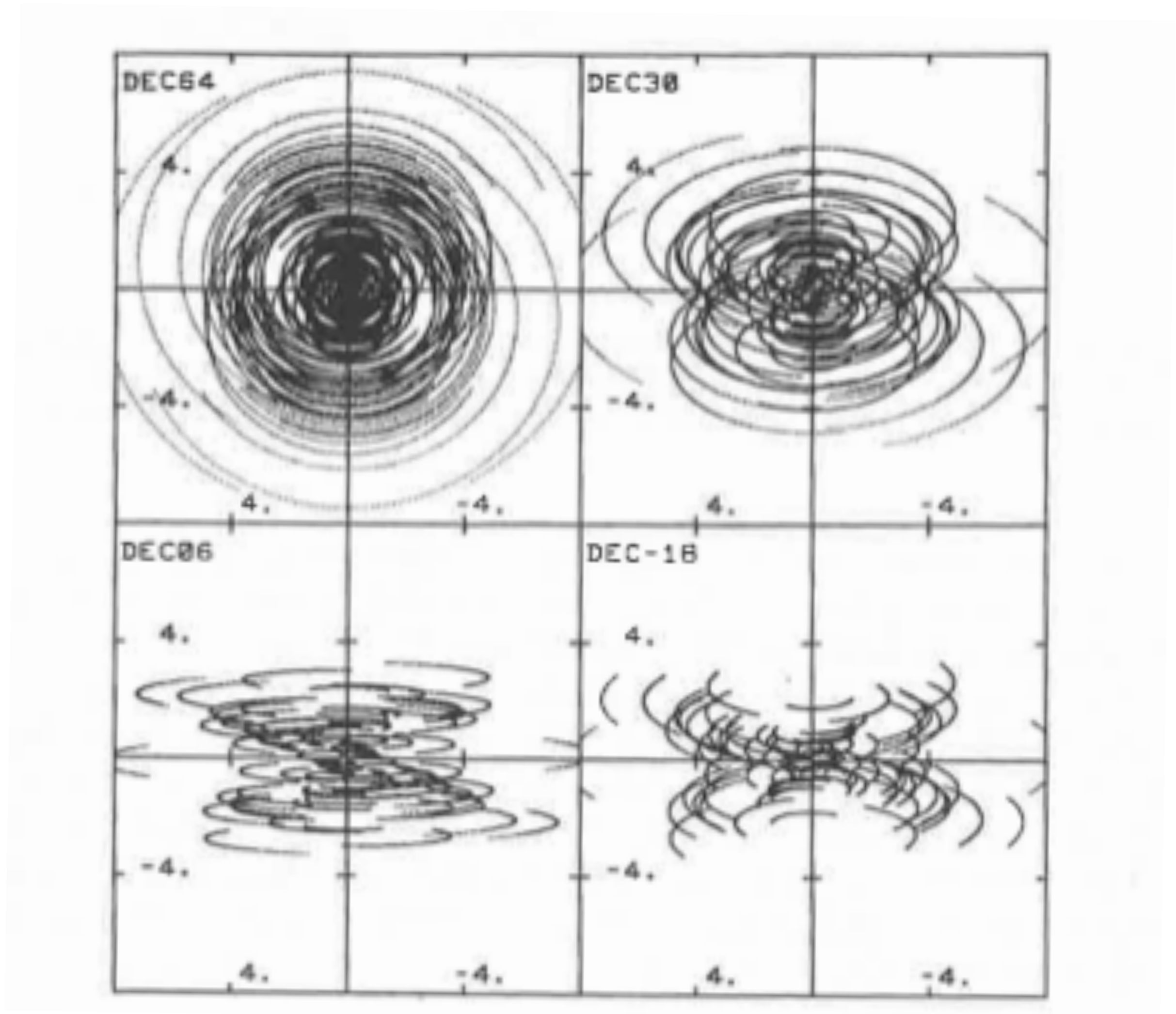
Figure 2–14. (a) The configuration of the 27 antennas of the VLA. (b) The transfer functions for four declinations with observing durations of $\pm 4^h$ for $\delta = 0°$ and 45°, $\pm 3^h$ for $\delta = -30°$, and $\pm 5^m$ for the snapshot. [From Napier, Thompson & Ekers (© 1983 IEEE).]

(Thompson, Moran & Swenson)

(u,v) -plane



(u,v) tracks as ellipses



Holes correspond to missing information.

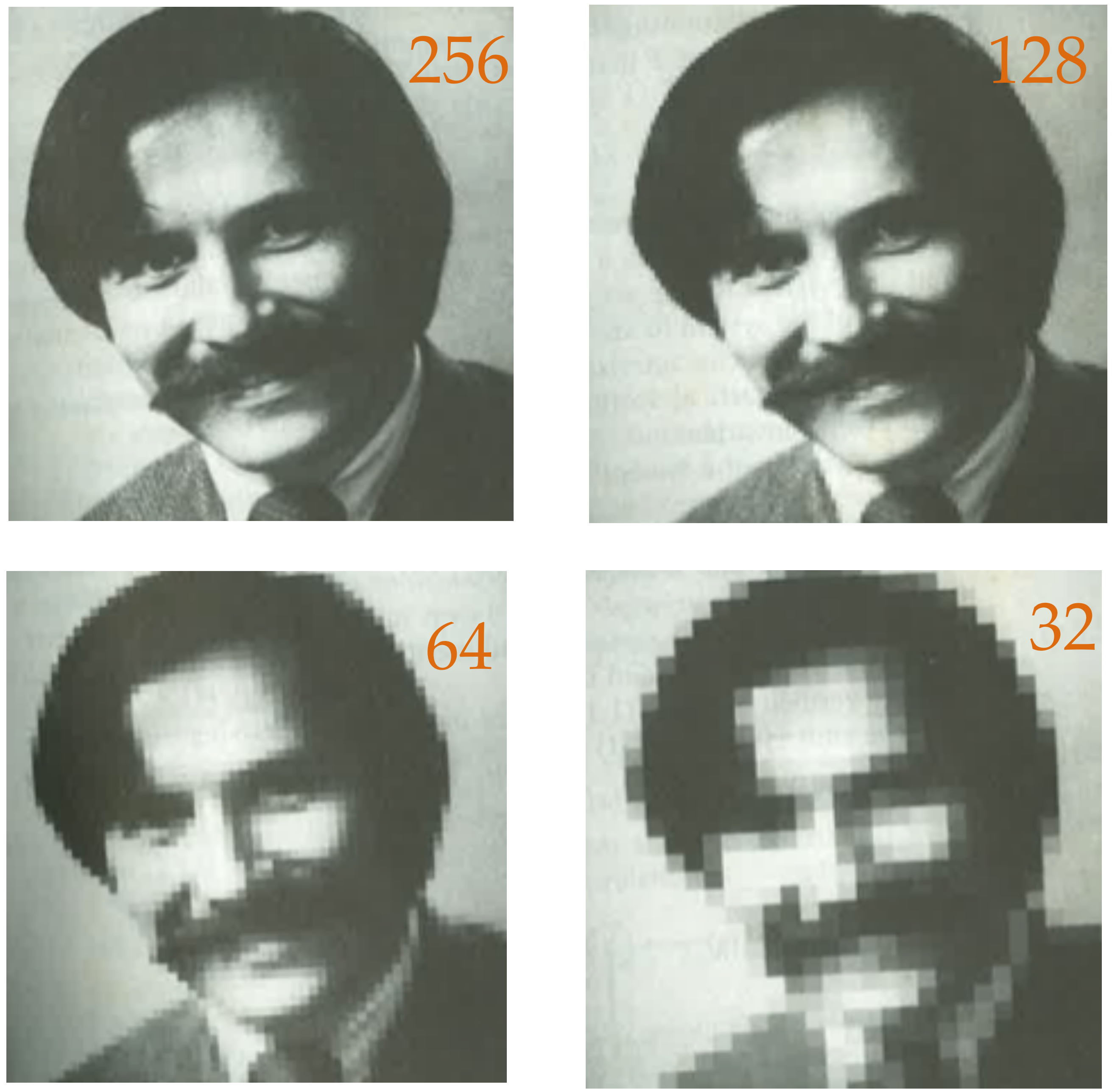
(Thompson, Moran & Swenson)

Image reconstruction: phase vs amplitude

Digital images: Quantization

Lim (1990)

Digital images: Pixelization

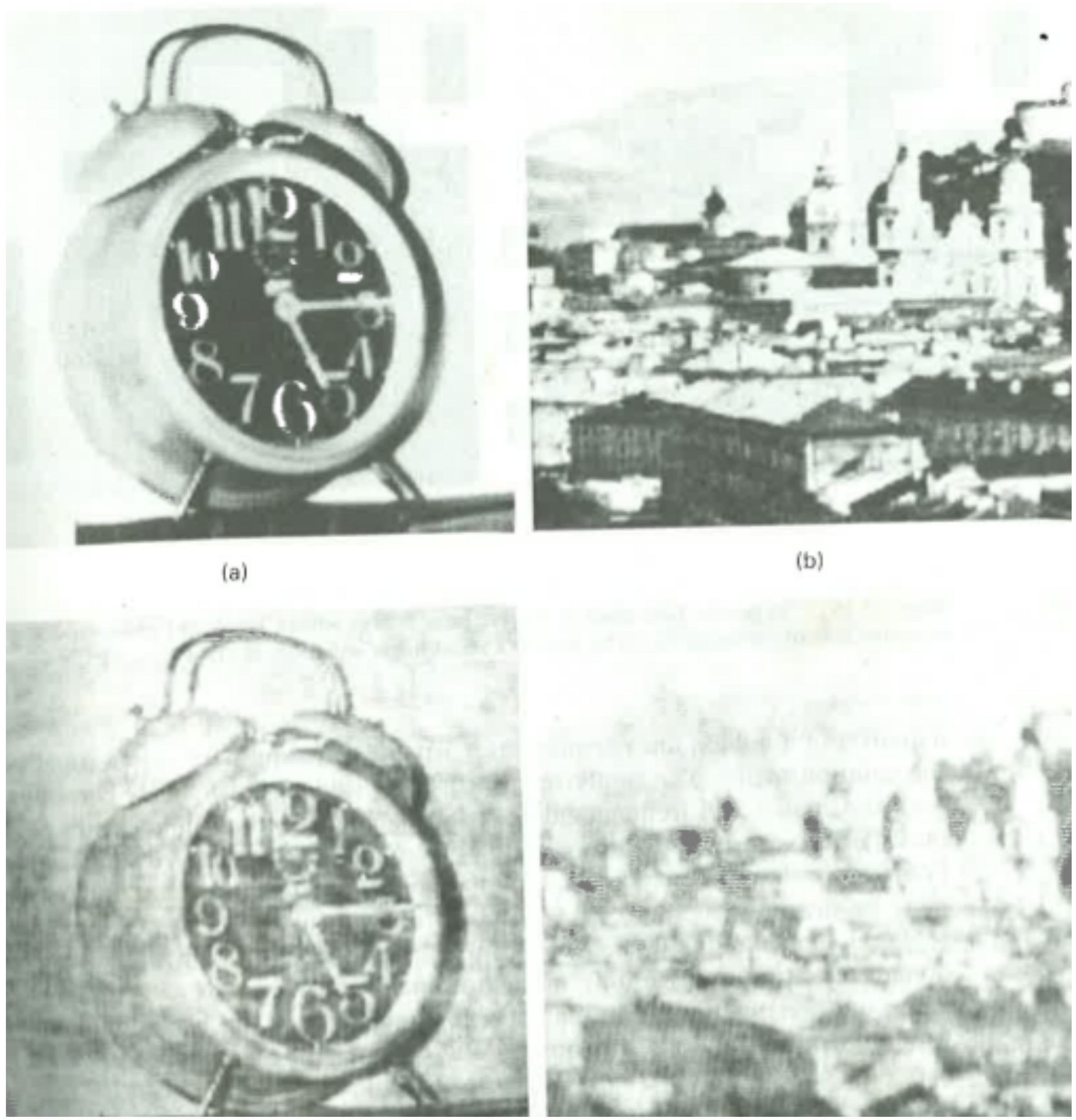


Lim (1990)

Phase/amplitude-only synthesis

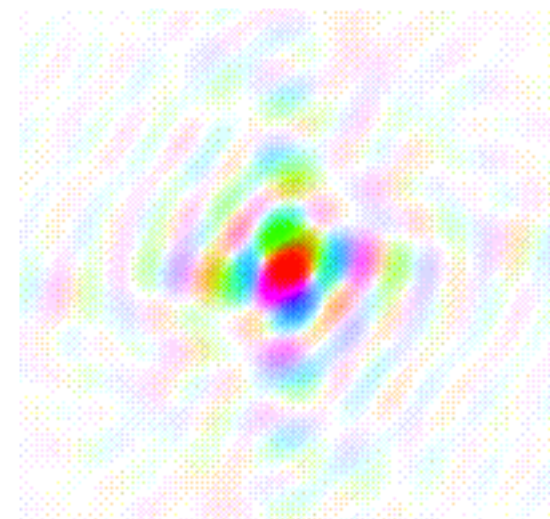
Lim (1990)

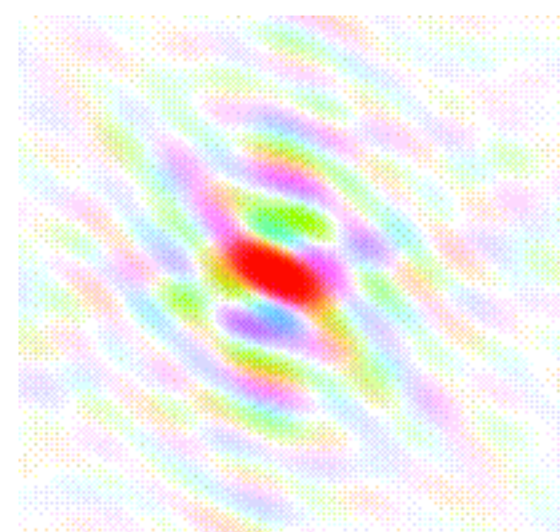
Phase/amplitude-only synthesis



Lim (1990)

Phase/amplitude-only synthesis

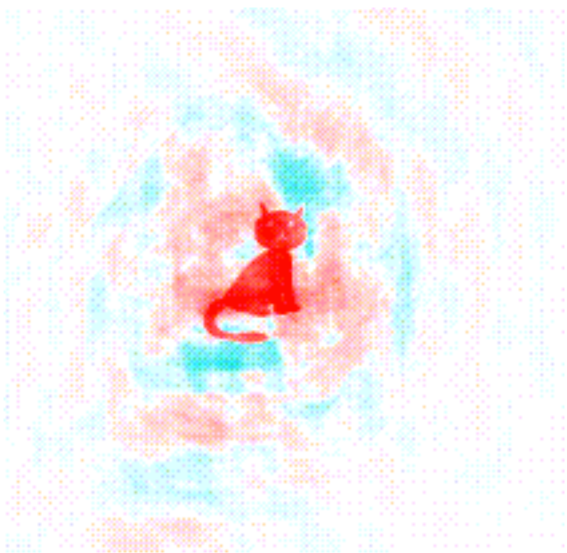




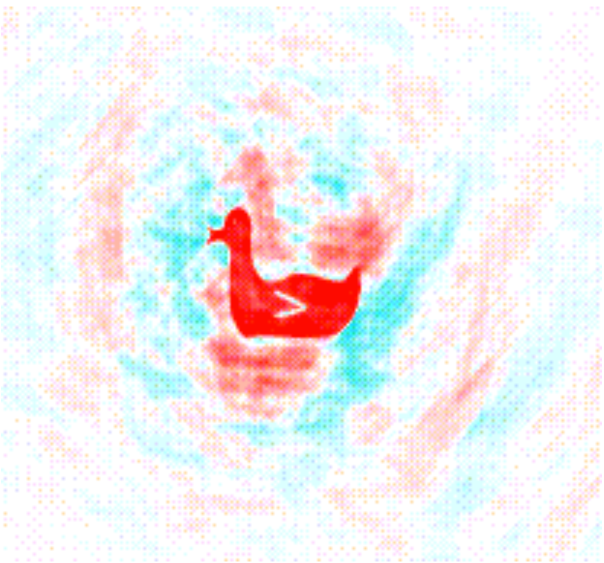
(Taylor, C. A. & Lipson, H., Optical Transforms, Bell, London 1964)

http://www.ysbl.york.ac.uk/~cowtan/fourier/magic.html

Phase/amplitude-only synthesis



Duck (Amplitude) + Cat (Phase)



Cat (Amplitude) + Duck (Phase)

(Taylor, C. A. & Lipson, H., Optical Transforms, Bell, London 1964)

http://www.ysbl.york.ac.uk/~cowtan/fourier/magic.html

Amplitude: magnitude of the spatial frequency.
Phase: it's location.

Image analysis

Lim (1990)

Image reconstruction: a few components required

12%

5%

Lim (1990)

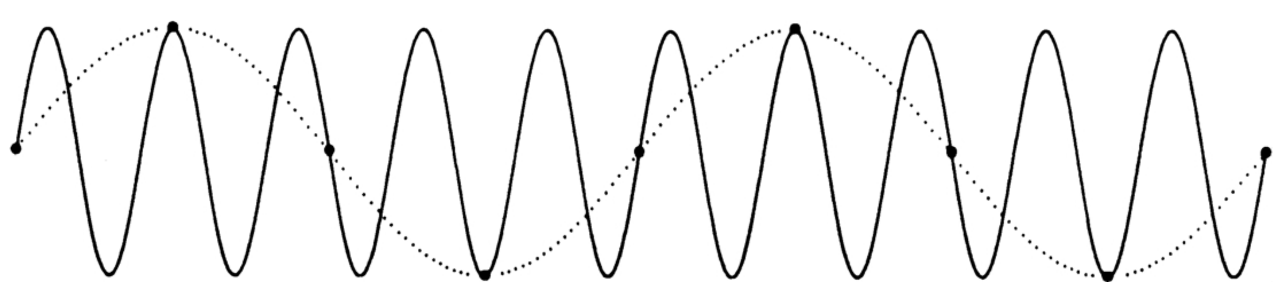
Sampling V(u,v)

Nyquist rate

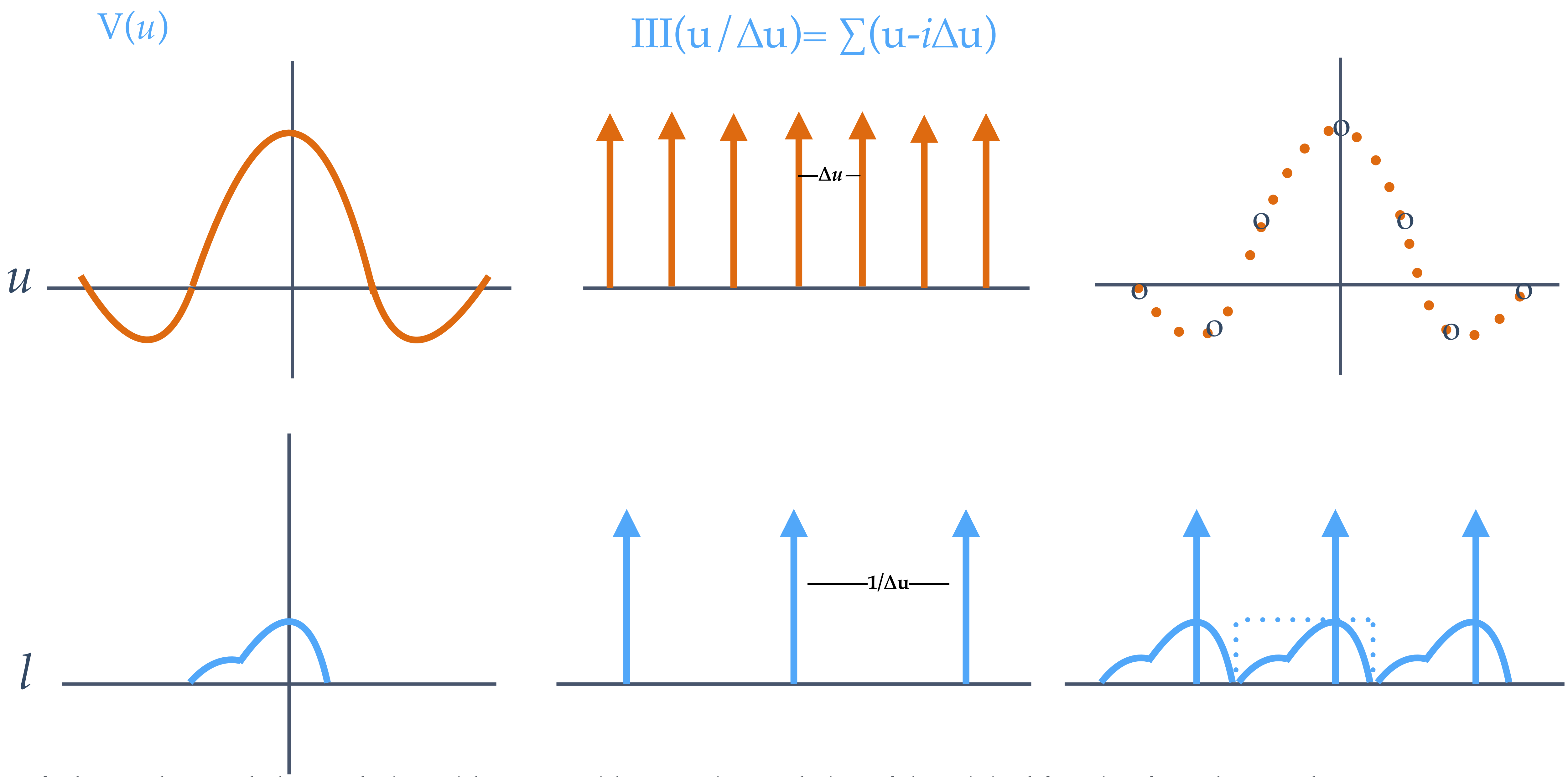
Any continuous band-limited signal can be reconstructed if sampled at the Nyquist rate.

Sampling rate = 1/2f

Higher frequency components will be aliased to the lower frequencies in the sampled band.



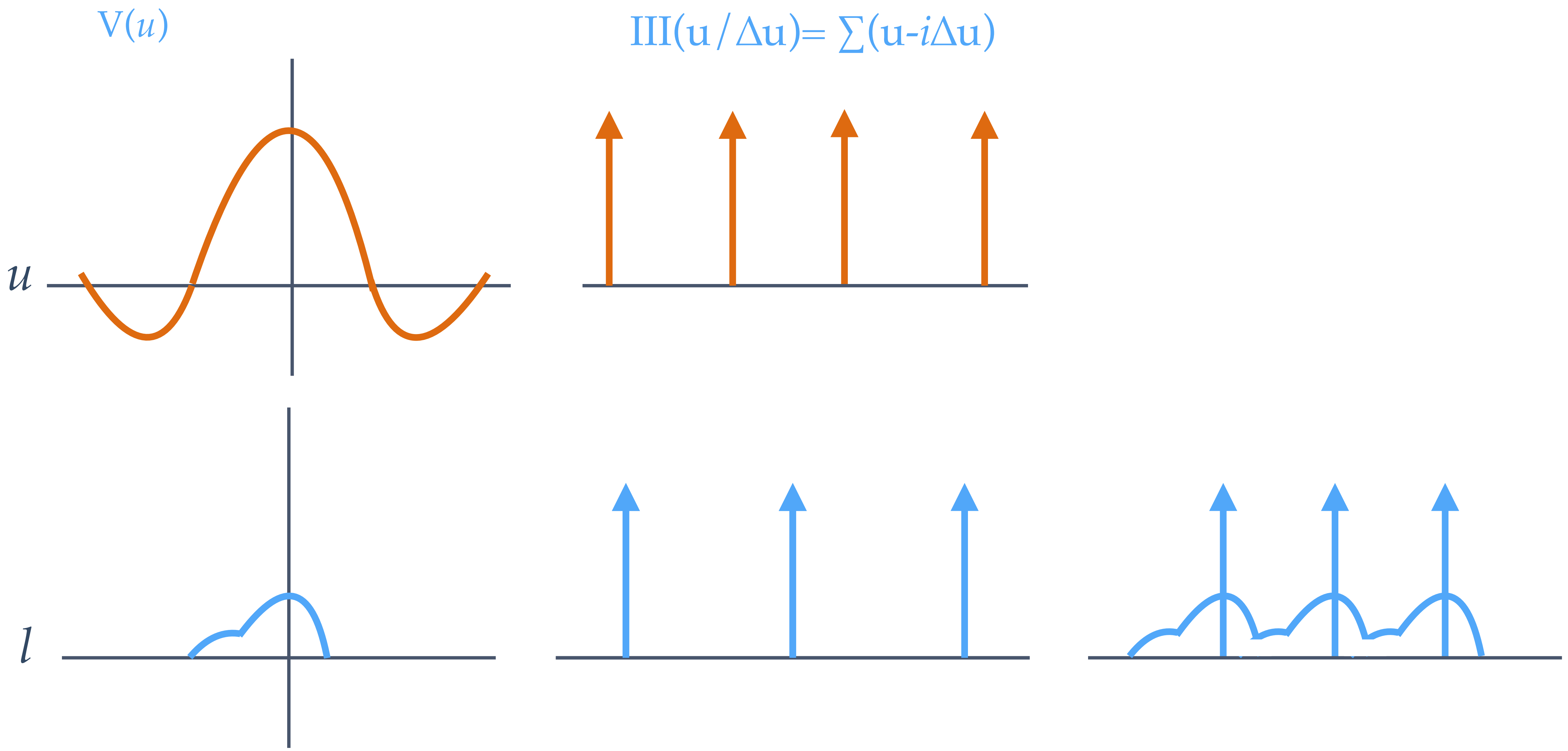
Sampling V(*u*,*v*)



<u>If adequately sampled convolution with *sinc* provides exact interpolation of the original function from the samples.</u>

(Thompson, Moran & Swenson)

Under-Sampling: Aliasing

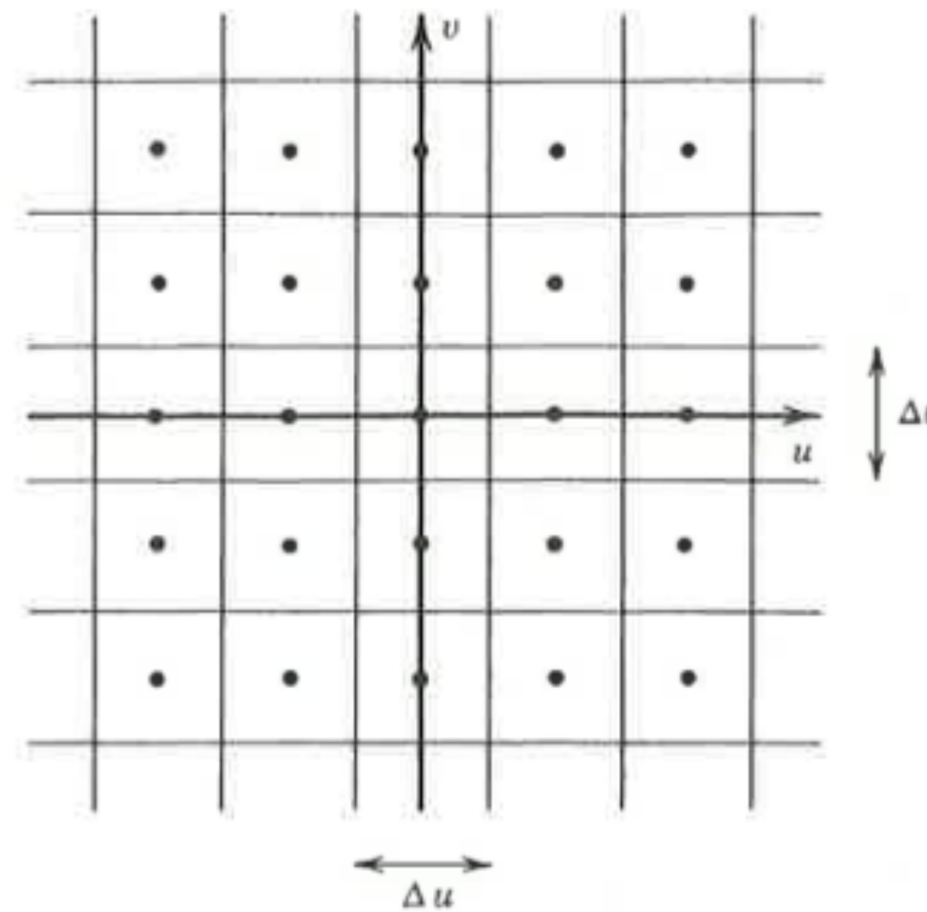


If aliasing is avoided convolution with *sinc* provides exact interpolation of the original function from the samples.

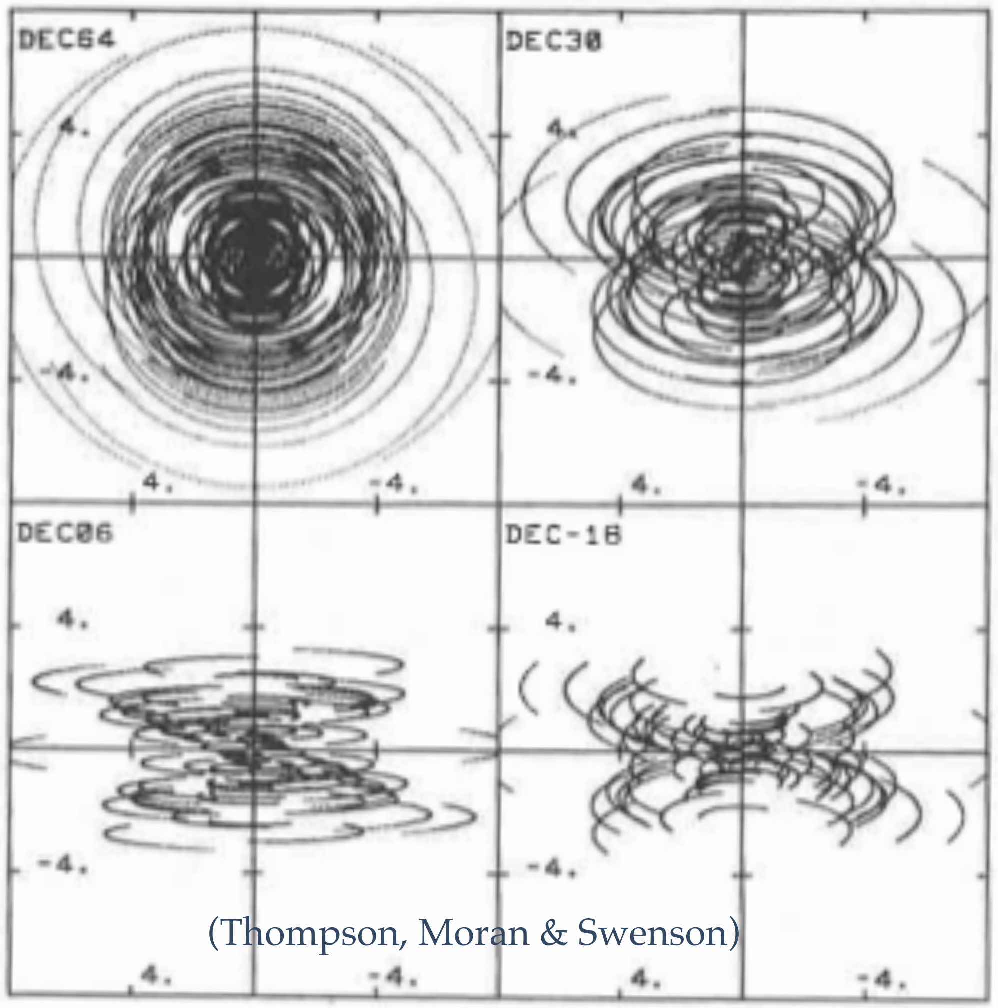
(Thompson, Moran & Swenson)

Fast Fourier Transform: V(u, v) - I(l,m)

- Faster.
- Requires data on uniform grid.
- Gridding to resample V(u,v).



Fast Fourier Transform: Image domain

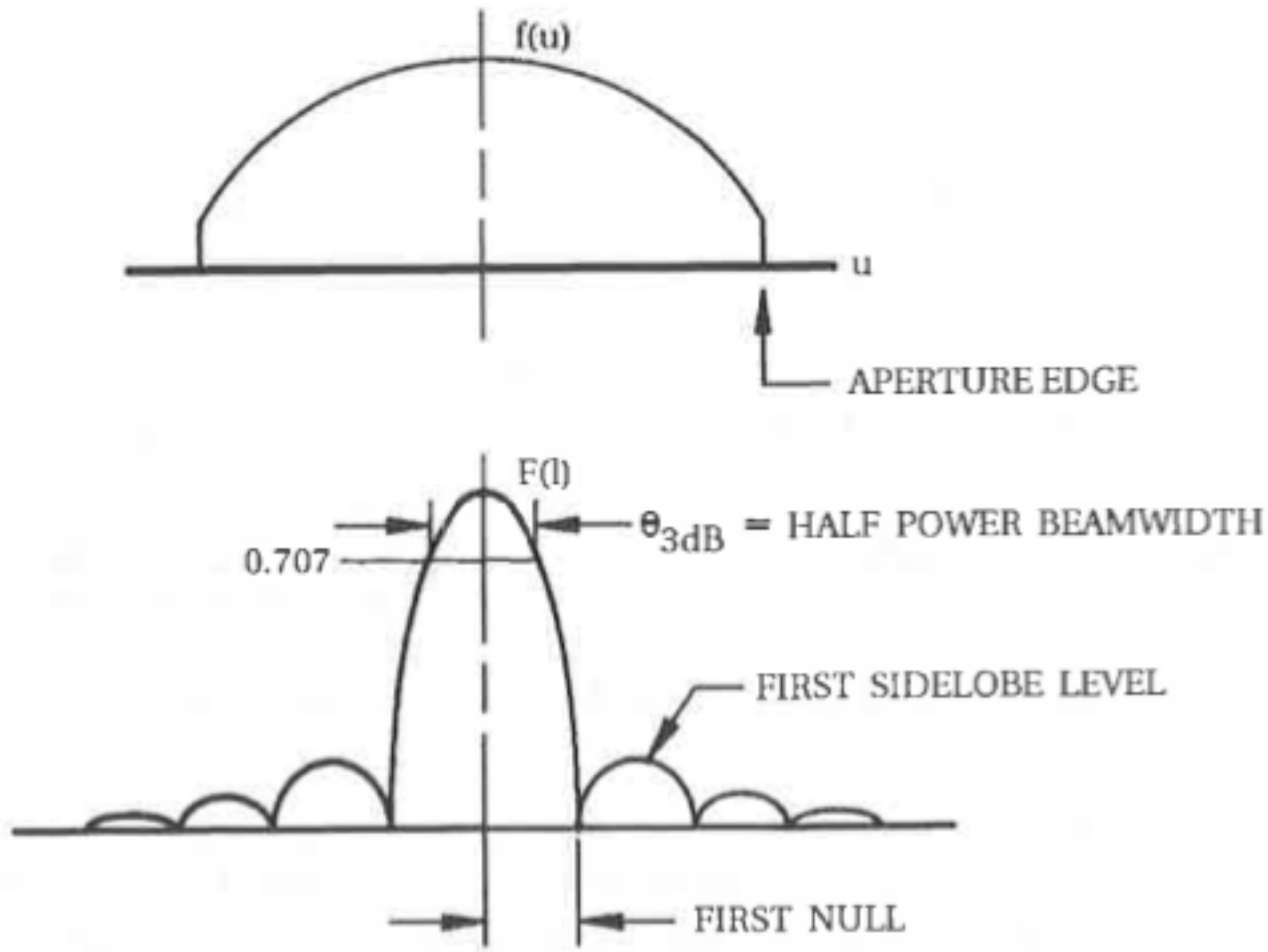


(Thompson, Moran & Swenson)

- Holes correspond to missing information.
- Longest baseline: limit on resolution
- Inner hole: no information on large scales

- Pixel size: $1/(2u_{max})$, $1/(2v_{max})$ i.e. satisfy sampling theorem.

Fast Fourier Transform: Image domain

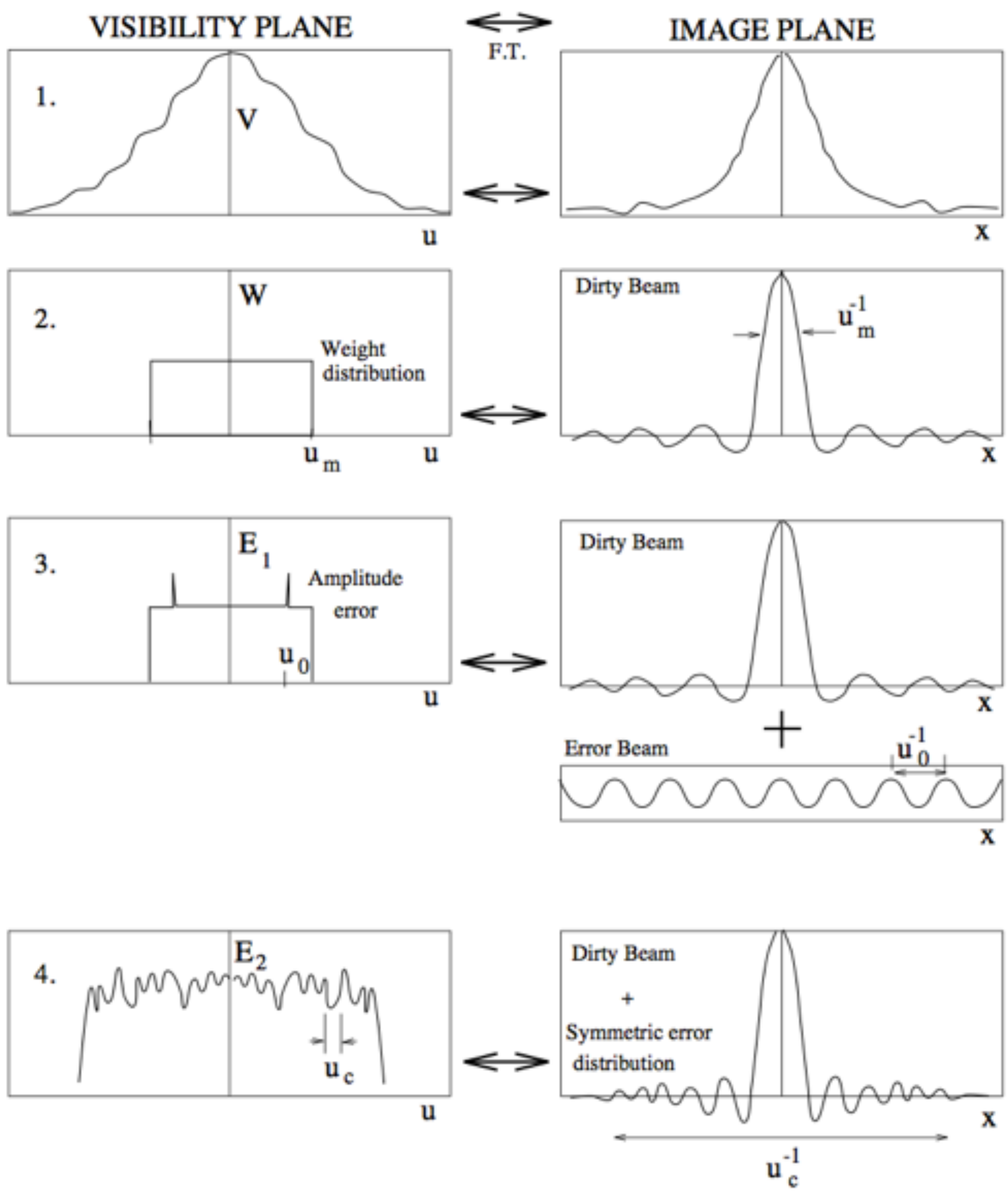


- Image size: whole primary beam; sources in the side lobe will be aliased back. Solution: make larger image !

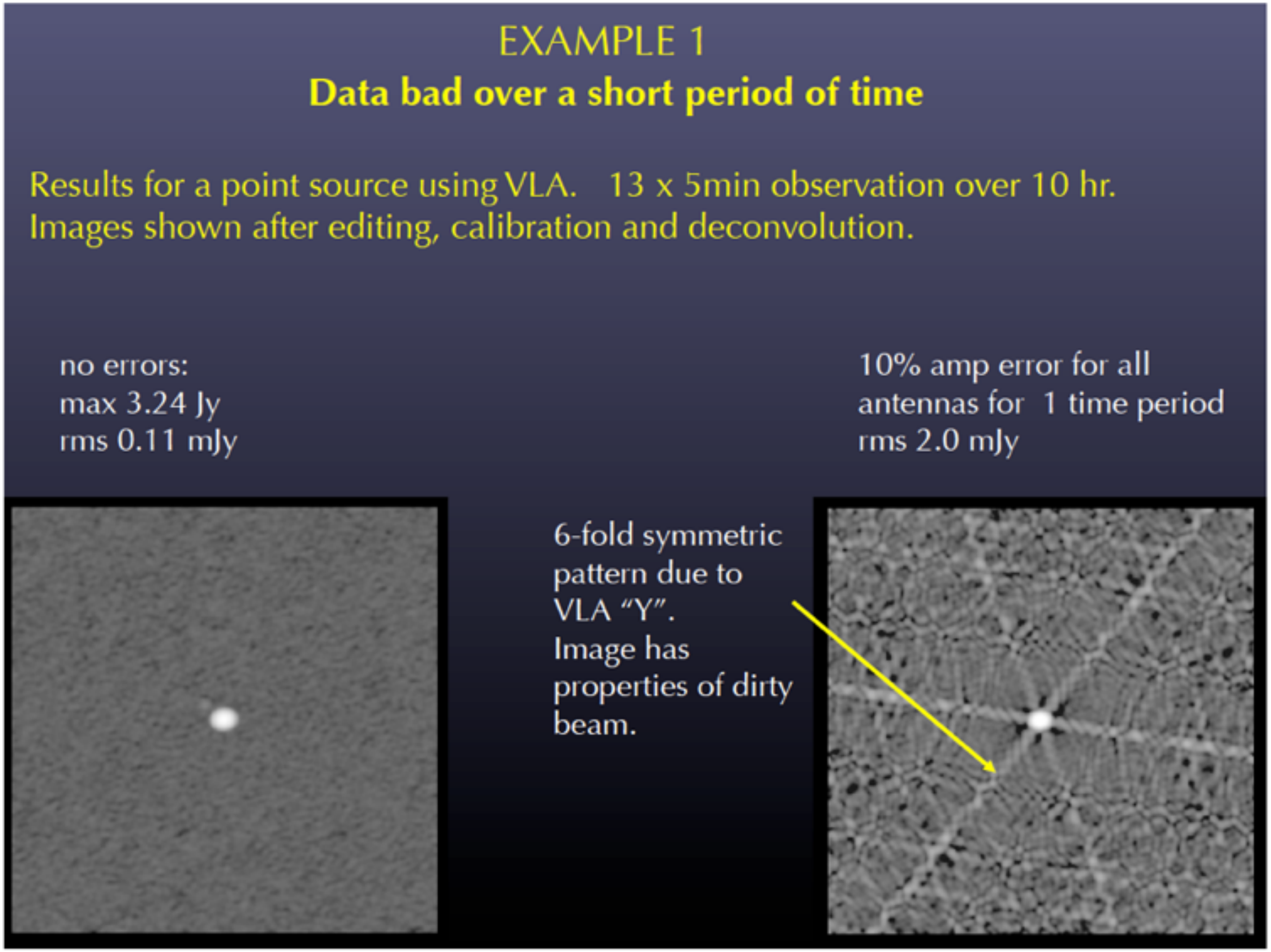
(Thompson, Moran & Swenson)

Errors in V(u,v)

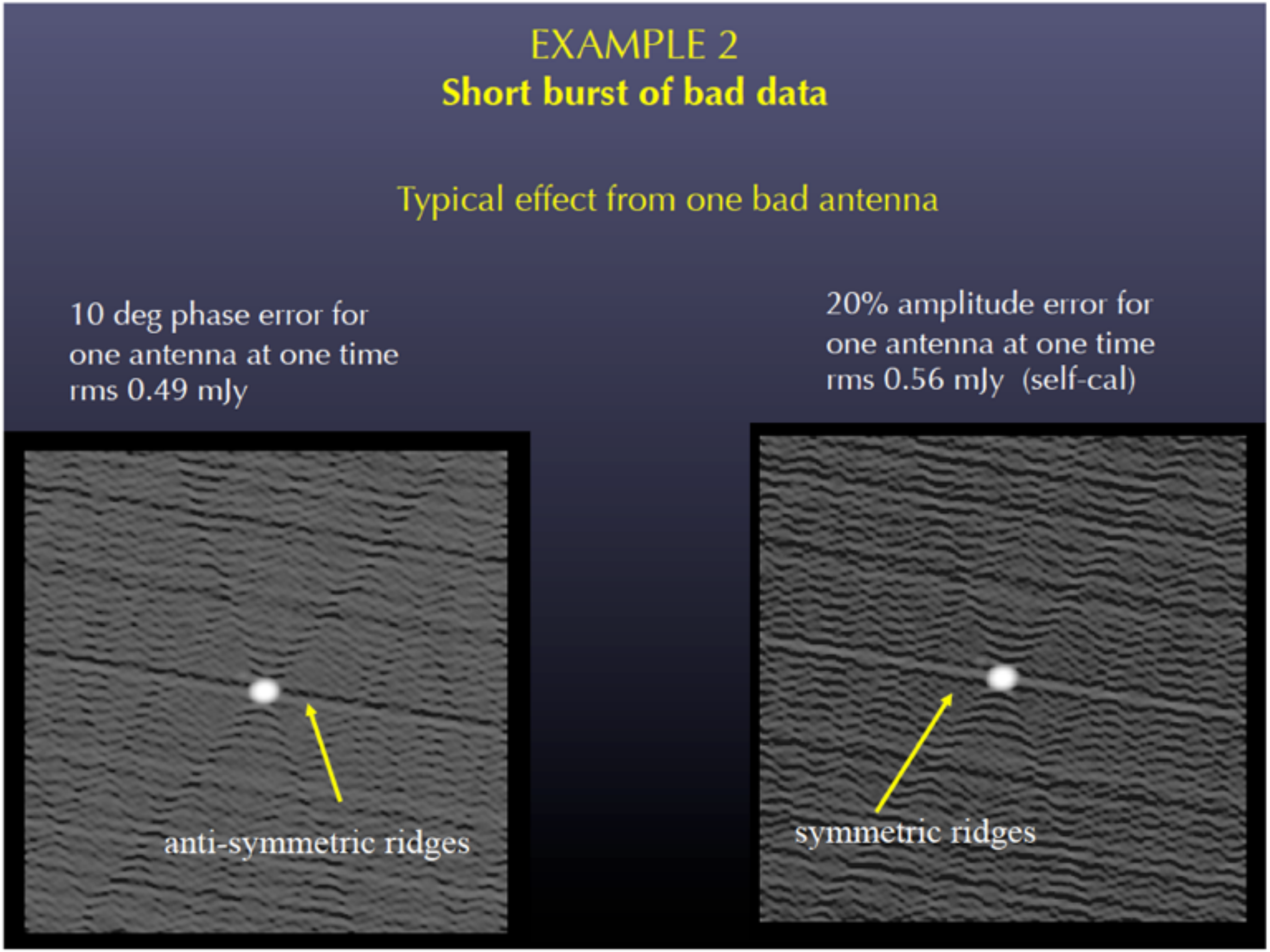
Effect of Amplitude error



(Thomson, Moran & Swenson)



Taylor et al. lecture (NRAO Synthesis Imaging School 2012)



Taylor et al. lecture (NRAO Synthesis Imaging School 2012)

$$V'(obs) = G_{ij}\, V(true)$$

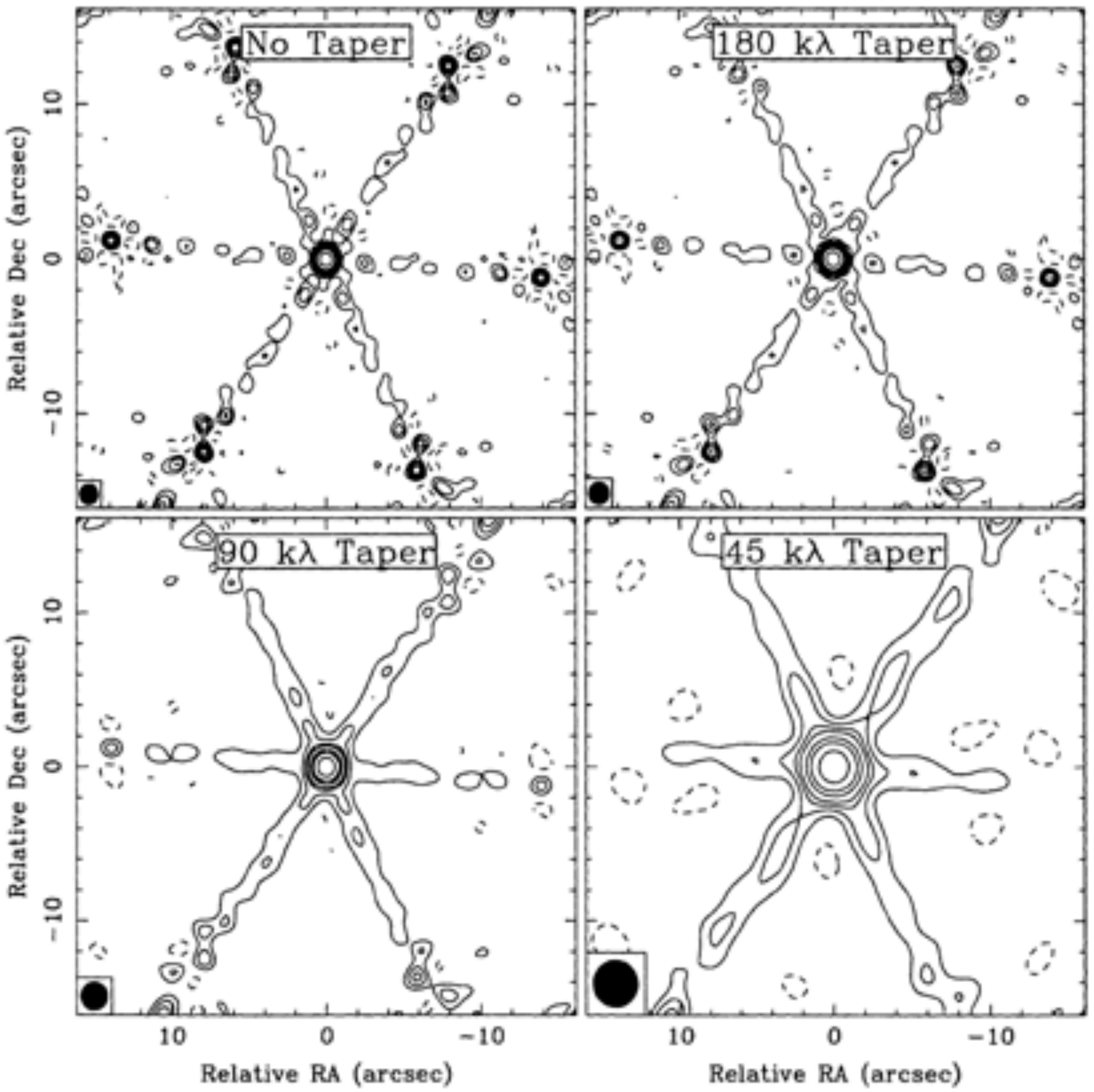
-Observing set-up
-bad data

poor calibration/
baseline-based errors

Diffuse extended emission

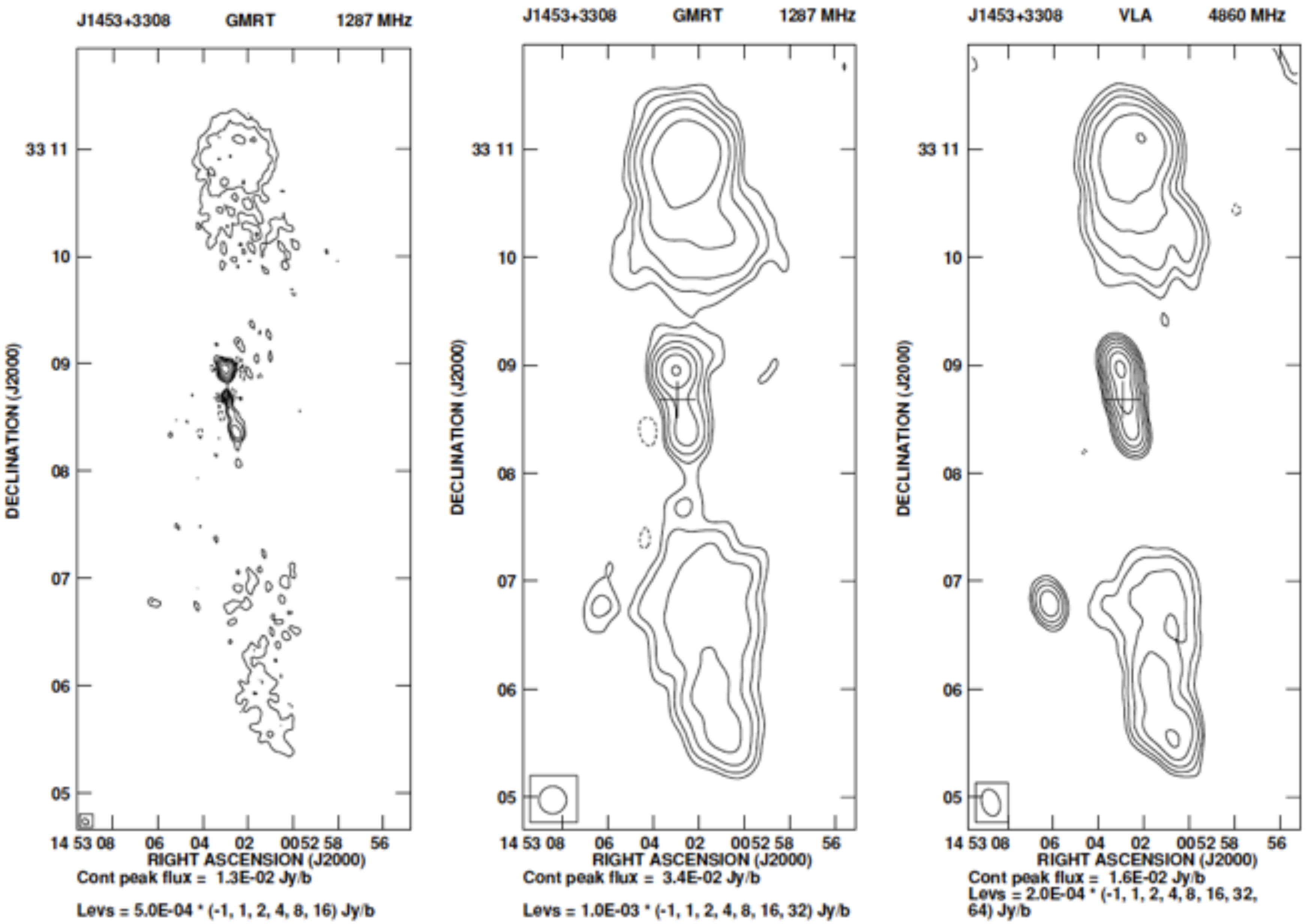
1) Weighting: surface brightness sensitivity
2) Masking: deconvolution & flux density

(Flux calculated correctly for cleaned map.)



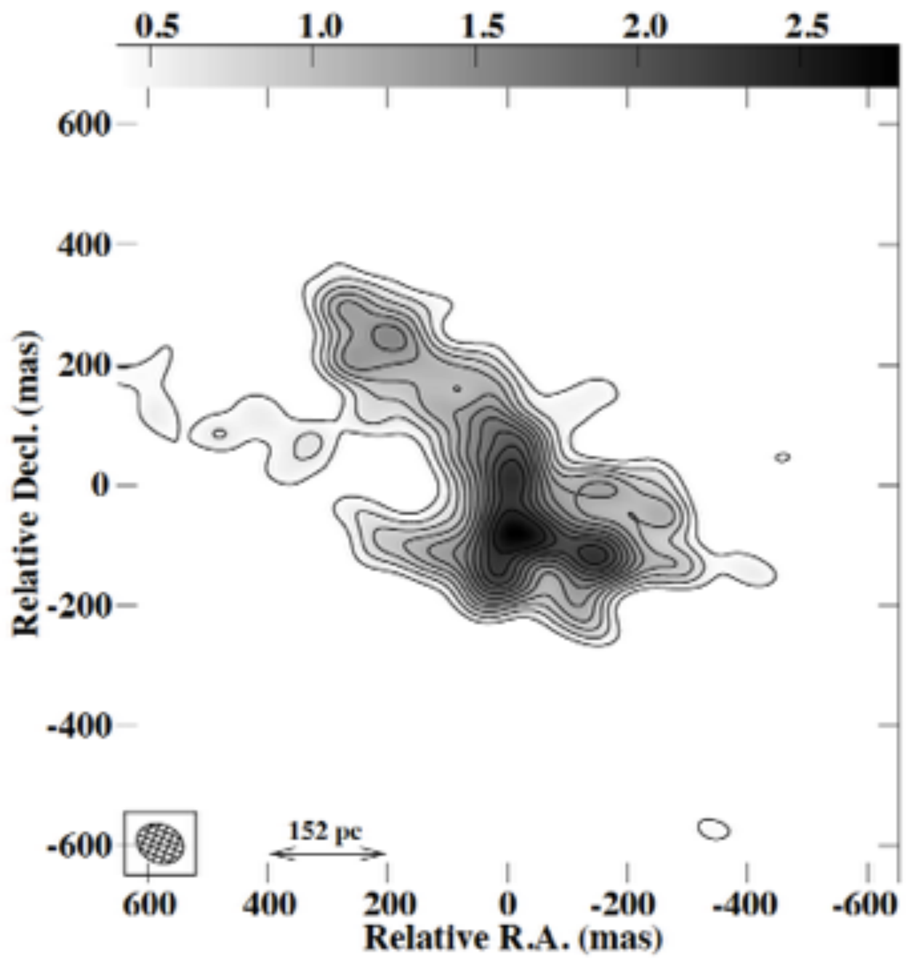
(Briggs et al. 1999)

Emission at various scales



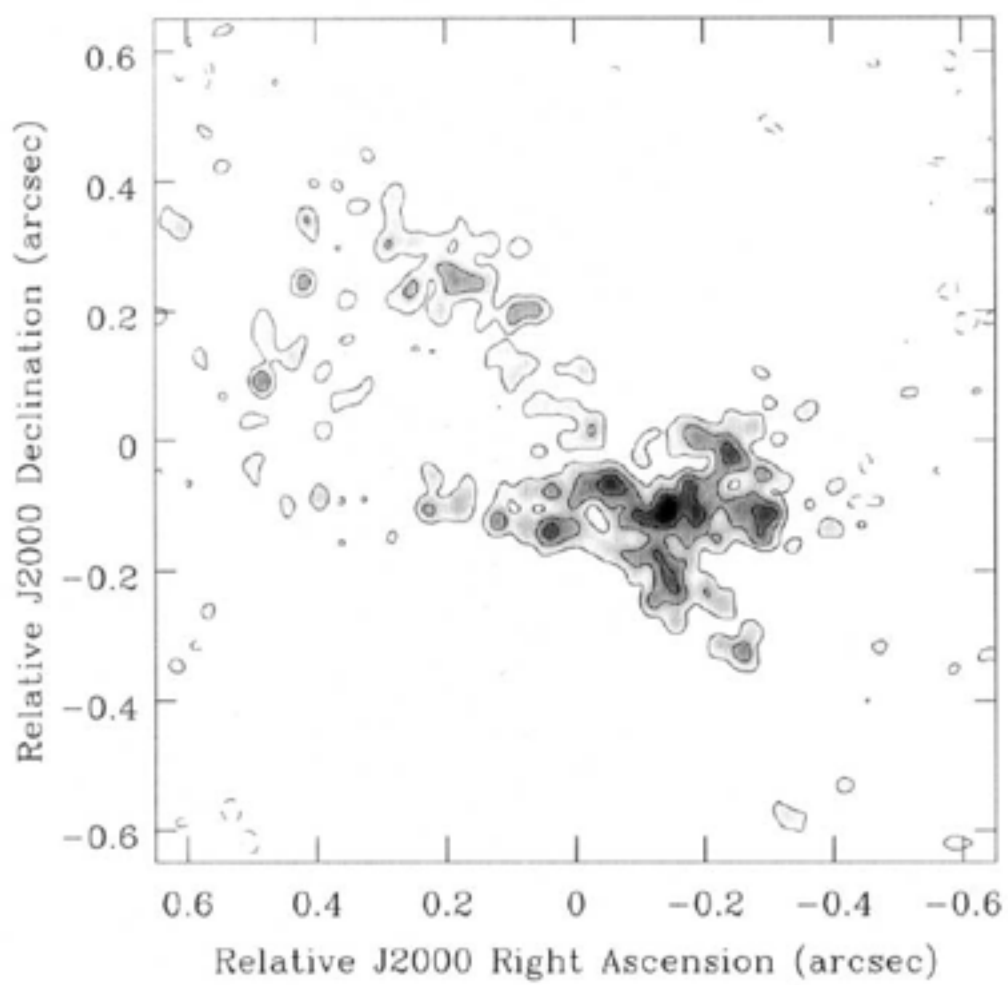
(Konar et al. 2006)

Emission at various scales



80x63 mas

Heavier Gaussian taper



36x33 mas

Momjian et al. 2003

Summary

- V(u,v) $\longleftrightarrow$ I(l,m)
- Radio interferometer samples V(u, v): fourier transform to get image.
- Fourier transforms also useful in identifying problems.
- Use Flagging, Gridding and Weighting of the visibility to get *appropriate* image.

References and further reading

- **Bracewell**: The Fourier Transform and its applications.
- **Thompson, Moran & Swenson**: Interferometry and Synthesis in Radio Astronomy.
- **Synthesis Imaging in Radio Astronomy II**: the NRAO lecture series.

END - PART II